Chapter 8 - Rate of Return Analysis: Multiple Alternatives

Why incremental analysis?

- In some cases, ROR analysis does not provide the same ranking of alternatives as do PW or AW analyses.
- Using incremental cash flow ROR analysis will try to avoid ranking inconsistency.

Example: MARR = 10% per year

<table>
<thead>
<tr>
<th></th>
<th>Alt A</th>
<th>Alt B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cost</td>
<td>$100</td>
<td>$100</td>
</tr>
<tr>
<td>Revenue at the end of life</td>
<td>$116</td>
<td>$201.14</td>
</tr>
<tr>
<td>Life</td>
<td>1 year</td>
<td>5 years</td>
</tr>
</tbody>
</table>

\[
\text{PW Analysis (using LCM of lives = 5 years):} \\
- \text{One cycle of A: } -100 + 116 \times (P/F,10\%,1) = 5.45 \\
- \text{PW}_A = 5.45 + 5.45 \times (P/A,10\%,4) = 22.74 \\
- \text{PW}_B = -100 + 201.14 \times (P/F,10\%,5) = 24.89 \quad \text{will choose B}
\]

\[
\text{ROR Analysis: (using LCM of 5 years)} \\
- ROR_A: \quad 0 = -100 + 16 \times (P/A,i^*\%,5) + 100 \times (P/F,i^*\%,5) \\
- \text{solving for } i^* \text{ we get } i^* = 16\% \\
- ROR_B: \quad 0 = -100 + 201.14 \times (P/F,i^*\%,5) \\
- \text{solving for } i^* \text{ we get } i^* = 15\% \quad \text{will choose A}
\]

Why incremental analysis?

- We have inconsistent rankings. Why?
  - PW assumes reinvestment at the MARR (or given interest rate)
  - ROR assumes reinvestment at the ROR, \( i^* \)
  - So we have two different reinvestment assumptions

Use incremental cash flow for ROR analysis.
### Example 8.1:

**Machine A**
- Initial cost: $15,000
- Operating cost: $8200/yr
- 5% salvage value
- 25-year life

**Machine B**
- Initial cost: $21,000
- Operating cost: $7000/yr
- 5% salvage value
- 25-year life

Create the Incremental Cash Flow Table.

### Example 8.1, Solution:

<table>
<thead>
<tr>
<th>Year</th>
<th>Machine A</th>
<th>Machine B</th>
<th>Inc. Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-15,000</td>
<td>-21,000</td>
<td>-6,000</td>
</tr>
<tr>
<td>1-25</td>
<td>-8,200</td>
<td>-7,000</td>
<td>1,200</td>
</tr>
<tr>
<td>25</td>
<td>750</td>
<td>1,050</td>
<td>300</td>
</tr>
<tr>
<td>Total</td>
<td>-219,250</td>
<td>-194,950</td>
<td>24,300</td>
</tr>
</tbody>
</table>

### Example 8.2:

**Type A**
- Initial cost: $70,000
- AOC: $9000
- Salvage Value: $5000
- 8-year life

**Type B**
- Initial cost: $95,000
- AOC: $7000
- Salvage Value: $10,000
- 12-year life

Create the Incremental Cash Flow Table.

### Example 8.3 (MARR = 10%)

<table>
<thead>
<tr>
<th>Year</th>
<th>A</th>
<th>B</th>
<th>B-A</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-30,000</td>
<td>-50,000</td>
<td>-20,000</td>
</tr>
<tr>
<td>1</td>
<td>-15,000</td>
<td>10,000</td>
<td>25,000</td>
</tr>
<tr>
<td>2</td>
<td>20,000</td>
<td>30,000</td>
<td>10,000</td>
</tr>
<tr>
<td>3</td>
<td>40,000</td>
<td>25,000</td>
<td>-15,000</td>
</tr>
<tr>
<td>4</td>
<td>5,000</td>
<td>-2,000</td>
<td>-7,000</td>
</tr>
</tbody>
</table>

$PW_A = $6,360.22$, $ROA = 16.36\%$

$PW_B = $1,301.14$, $ROB = 11.33\%$

If $ROA > MARR$ and $PWB > PW_A$, then Type B is preferable.

Conclusions:

- $ROA > MARR$, so Type A is feasible.
- $PWB > PW_A$, so Type B is feasible.
- Select Type B.
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**Incremental ROR by PW**

**INDEPENDENT** - Selection of one alternative does not affect the selection of others.

**MUTUALLY EXCLUSIVE** - Selection of one alternative precludes the selection of others.

**Incremental ROR by PW Equation**

- Mutually Exclusive Alternatives:
  - Required elements
    1. Incremental cash flow series
    2. LCM of lives
  - It is generally advisable to use PW or AW at MARR when multiple rates are indicated.
  - If \( \Delta^b-A < MARR \) then select A, else select B.

- Independent Projects:
  - No incremental investment comparison needed. Select all projects with ROR \( \geq \) MARR.

**Incremental ROR by PW Procedure**

1. Order the alternatives by initial investment cost with the larger one being alternative B.
2. Develop the cash flow and incremental cash flow series over the LCM of lives.
3. Draw an incremental cash flow diagram, if needed.
4. Count the number of sign changes in the incremental cash flow series to determine if multiple ROR may be present. If necessary, use Norstrom’s criterion on the cumulative incremental cash flow series to determine if a single positive root exists.
5. Set up the PW equation for the incremental cash flows and determine \( \Delta^b-A \).
6. Select the economically better alternative as follows:
   - If \( \Delta^b-A < MARR \), select alternative A.
   - If \( \Delta^b-A > MARR \), the extra investment is justified; select alternative B.

**Example 8.4:** Given the following information on Verizon’s choices for an equipment vendor, determine which vendor should be selected if MARR is 15% per year (Excel file ch8).

<table>
<thead>
<tr>
<th>Year</th>
<th>CF A</th>
<th>CF B</th>
<th>Inc. CF</th>
<th>Cum. Inc. CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-8000</td>
<td>-13,000</td>
<td>-5000</td>
<td>-5000</td>
</tr>
<tr>
<td>1</td>
<td>-3500</td>
<td>-1600</td>
<td>1900</td>
<td>-3100</td>
</tr>
<tr>
<td>2</td>
<td>-3500</td>
<td>-1600</td>
<td>1900</td>
<td>-1200</td>
</tr>
<tr>
<td>3</td>
<td>-3500</td>
<td>-1600</td>
<td>1900</td>
<td>700</td>
</tr>
<tr>
<td>4</td>
<td>-3500</td>
<td>-1600</td>
<td>1900</td>
<td>2600</td>
</tr>
<tr>
<td>5</td>
<td>-3500</td>
<td>-1600</td>
<td>1900</td>
<td>4500</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
<td>-</td>
<td>2000</td>
<td>-13,000</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>-</td>
<td>-11,000</td>
<td>-6500</td>
</tr>
<tr>
<td>8</td>
<td>-</td>
<td>-</td>
<td>-6400</td>
<td>-11,000</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
<td>-6400</td>
<td>-6500</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
<td>-</td>
<td>-43,000</td>
<td>-10000</td>
</tr>
<tr>
<td>Year</td>
<td>-43,000</td>
<td>-38,000</td>
<td>5000</td>
<td>10000</td>
</tr>
</tbody>
</table>

**Example 8.4, Solution:**

Number of roots analysis:

**Descarte’s rule:**
Number of sign changes: 3 ➔ At most 3 roots

**Norstrom’s rule:** Number of sign changes of cumulative incremental cash flow: 3 ➔ so not applicable.
Example 8.4, Solution (continued):
Use the ROR equation based on PW of the incremental cash flow.

\[ 0 = -5000 + 1900(P/A, \Delta i, 10) - 11,000(P/F, \Delta i, 5) + 2000(P/F, \Delta i, 10) \]

Assume the reinvestment rate is equal to the resulting \(\Delta i_{B-A}\).

Solve for \(\Delta i\) using trial/error, etc. \(\Delta i = 12.65\%\)

Note: In theory, there are three roots to the equation, however, there is only one (12.65%) falling in the practically reasonable range. For example, the other one is \(-216\%\), falling out of the range of acceptable range of \(i^*\) (-100%, +infinity).

Since the ROR of 12.65% on the extra investment is less than the 15% MARR, the lower-cost vendor A is selected.

Example 8.4, Solution (continue):
Breakeven ROR Interpretation
For MARR<12.65%, vendor B is justified.
For MARR>12.65%, vendor A is selected.
If MARR=12.65%, the alternatives are equally attractive.

This analysis assumes that \(PWB_A(i) > 0\) for \(i < \Delta i_{B-A}\) and \(PWB_A(i) < 0\) for \(i > \Delta i_{B-A}\) so that ROR analysis is consistent with PW analysis at MARR.

Example 8.5
IBM is considering buying a chip placement machine. The machine cost 260K with annual operating expense 25K and a salvage value of 25K after 10 years. A slightly used machine is available for 150K with annual operating expense of 40K and a salvage value of 20K after 10 years. Which machine should be selected if MARR is 10% per year.

Example 8.5, Solution: PW Analysis

<table>
<thead>
<tr>
<th>year</th>
<th>Alt A</th>
<th>Alt B</th>
<th>B-A</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-150K</td>
<td>-260K</td>
<td>-110K</td>
</tr>
<tr>
<td>1-10</td>
<td>-40K</td>
<td>-25K</td>
<td>15K</td>
</tr>
<tr>
<td>10</td>
<td>20K</td>
<td>25K</td>
<td>5K</td>
</tr>
</tbody>
</table>

\[ PW = \frac{-110}{P/A, 0.1, 10} + 5(P/F, 0.1, 10) \]
\[ = \frac{-110}{1.0} + 5*0.67 < 0 \]

additional investment is not justified
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Conclusion:

If the ROR method is used to evaluate two or more alternatives, use the incremental cash flow and \( \Delta i^* \) to make the decision between alternatives.

ROR evaluation by AW

Using the AW relation, there are two equivalent ways for ROR analysis:

1. Use incremental cash flows over the LCM of alternative lives

\[
0 = AW_B - AW_A
\]

OR

2. Use actual cash flows for one life cycle,

\[
0 = AW_B - AW_A
\]

Example 8.6: Two machines are being considered for purchase. If MARR is 10%, which machine should be bought?

<table>
<thead>
<tr>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>$200</td>
</tr>
<tr>
<td>Annual benefit</td>
<td>95</td>
</tr>
<tr>
<td>Salvage value</td>
<td>50</td>
</tr>
<tr>
<td>Useful life, in years</td>
<td>6</td>
</tr>
</tbody>
</table>

a) Use incremental cash flows with LCM of lives=12 years

\[
0 = AW_A = -500(A/P, i^*, 12) + 25 + 150(F/A, i^*, 6)(A/P, i^*, 12) + 100(A/F, i^*, 12)
\]

Solve for \( i^* \), \( i^* = 1.32\% \)

Select Machine A

b) Use actual cash flows for one life cycle.

\[
AW_A = -200(A/P, i^*, 6) + 95 + 50(A/F, i^*, 6)
\]

\[
AW_B = -700(A/P, i^*, 12) + 120 + 150(A/F, i^*, 12)
\]

Set

\[
0 = AW_A - AW_B
\]

And solve for \( i^* \), \( i^* = 1.32\% \)

Select Machine A
ROR evaluation by AW

Example 8.7: Given the following information on Verizon’s choices for an equipment vendor, determine which vendor should be selected if MARR is 15% per year.

<table>
<thead>
<tr>
<th>Vendor</th>
<th>Initial Cost</th>
<th>Annual Costs</th>
<th>Salvage Value</th>
<th>Life, years</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-8,000</td>
<td>-3,500</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>-13,000</td>
<td>-1,600</td>
<td>2,000</td>
<td>5</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
&\text{AW}_A = -8,000(A/P,i,10) - 3,500 \\
&\text{AW}_B = -13,000(A/P,i,5) + 2,000(A/F,i,5) - 1,600 \\
&\text{AW}_B - \text{AW}_A = 0 \\
&-13,000(A/P,i,5) + 2,000(A/F,i,5) + 8,000(A/P,i,10) + 1,900 = 0
\end{align*}
\]

Using trial and error, \(i^* = 12.65\% < \text{MARR}\)

Select vendor A.

Multiple Alternatives (more than two)

- Compute \(i^*\) for each alternative individually and discard those with \(i^* < \text{MARR}\).
- Do incremental pairwise comparison for the accepted alternatives in order of smallest-to-largest initial investment.

Multiple Alternatives

Example 8.8: Four Alternative Building Locations

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Initial Cost</th>
<th>Annual CF</th>
<th>Life</th>
<th>MARR = 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-200,000</td>
<td>22,000</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>-275,000</td>
<td>35,000</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>-190,000</td>
<td>19,500</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>-350,000</td>
<td>42,000</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

Use incremental ROR AW analysis to select the best location.

Multiple Alternatives

Example 8.8, Solution:

1. The alternatives are ordered by increasing initial cost.
2. Compare C with DN (Do-Nothing).
\[
0 = -190,000 + 19,500(P/A, \Delta i^*, 30)
\]

\(\Delta i^* = 9.61\% < \text{MARR}\); Location C eliminated.

Now, compare A with DN.

\(\Delta i^* = 10.44\% > \text{MARR}\); DN eliminated.

Location A-defender; Location B-challenger.
Example 8.8, Solution (cont.):

3. Compare B with A.

\[ 0 = -275,000 + (35,000 - 22,000)(P/A, \Delta i^*, 30) \]
\[ \Rightarrow \frac{\Delta i^*}{\Delta i^*} = 17.18\% \text{, } > \text{MARR} \]
so Alternative B is justified incrementally; A eliminated.

4. Compare D with B.

\[ 0 = -75,000 + 7000(P/A, \Delta i^*, 30) \]
\[ \text{P/A value} = 10.7143, \Delta i^* = 8.53\% < \text{MARR} \]
Location D eliminated; only Location B remains.

Location B is selected.

---

Multiple Alternatives

Example 8.9: A pipeline is to be laid, the cost for the various size pipe are shown below. MARR is 8% and the life of the pipeline is 15 years.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cost</td>
<td>9,180</td>
<td>10,510</td>
<td>13,180</td>
<td>15,850</td>
<td>30,530</td>
</tr>
<tr>
<td>Installation</td>
<td>600</td>
<td>800</td>
<td>1,400</td>
<td>1,500</td>
<td>2,000</td>
</tr>
<tr>
<td>Ann. Cost</td>
<td>6,000</td>
<td>5,800</td>
<td>5,200</td>
<td>4,900</td>
<td>4,800</td>
</tr>
</tbody>
</table>

Example 8.9, Solution:

A vs. B (i.e B-A)

Initial Cost: -11,310 + 9,780 = -1,530

\[ 0 = 200(P/A, i^*, 15) - 1530 \]
\[ (P/A, .08, 15) = 8.56, \text{therefore, } i^* > 8\% \]

A is removed and B is the new defender

Example 8.9, Solution (continue):

B vs. C

Initial Cost: -14,580 + 13,180 = -3270

\[ 0 = 600(P/A, i^*, 15) - 3270 \]
\[ (P/A, .08, 15) = 5.45 < 8.56 = (P/A, .08, 15) \]

B is removed and C is the new defender

Example 8.9, Solution (continue):

C vs. D

Initial Cost: -17,350 - 2,770

\[ 0 = 300(P/A, i^*, 15) - 2770 \]
\[ (P/A, .08, 15) = 9.23 > 8.56 = (P/A, .08, 15) \]

D is removed and C is the defender
Example 8.9, Solution (continue):

C vs. E
Initial Cost 14,580 - 32,530 = -17,950
Ann. Cost 5200 - 4800 = 400

0 = 400 \( (P/A, i^*, 15) \) - 17,950
\( (P/A, i^*, 15) = 44.87 > 8.56 = (P/A, .08, 15) \)

\( C \) is the successful final alternative

---

Example 8.9, Solution (continue):

\[
\begin{align*}
\text{PW(A)} & = -9,780 - (P/A, .08, 15) 6,000 = -61,137 \\
\text{PW(B)} & = -11,310 - (P/A, .08, 15) 5,800 = -60,955 \\
\text{PW(C)} & = -14,580 - (P/A, .08, 15) 5,200 = -59,089 \\
\text{PW(D)} & = -17,350 - (P/A, .08, 15) 4,900 = -59,291 \\
\text{PW(E)} & = -32,530 - (P/A, .08, 15) 4,800 = -73,615 \\
\end{align*}
\]

Choose Largest PW, Alternative C