Chapter 7 - Rate of Return Analysis: Single Alternative

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Definition and Concept

Definition:
Rate of Return is either the "rate paid" on the unpaid balance of borrowed money, or the "rate earned" on the unrecovered balance of an investment, so that the final payment or receipt brings the balance to exactly zero with interest considered.

Example 7.1:
Consider the following loan
You lend $1,000 at 10% per year for 4 years.
Borrower makes 4 equal end-of-year payments to pay off this loan with interest considered.
The payments are:
A = $1,000(A/P,10%,4)= $315.47

At the end, the unpaid balance is zero

Computed at 10% on the beginning unpaid balance

<table>
<thead>
<tr>
<th>Year</th>
<th>Beginning unpaid balance</th>
<th>End-of-year payment</th>
<th>Interest amount 0.10*(col 2)</th>
<th>Recovered amount (col 3 + 4)</th>
<th>Ending unpaid balance (col 2 + 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1000.00</td>
<td>+315.47</td>
<td>-100.00</td>
<td>+215.47</td>
<td>-784.53</td>
</tr>
<tr>
<td>2</td>
<td>-784.53</td>
<td>+315.47</td>
<td>-78.45</td>
<td>+237.02</td>
<td>-547.51</td>
</tr>
<tr>
<td>3</td>
<td>-547.51</td>
<td>+315.47</td>
<td>-54.75</td>
<td>+260.72</td>
<td>-286.79</td>
</tr>
<tr>
<td>4</td>
<td>-286.79</td>
<td>+315.47</td>
<td>-28.68</td>
<td>+286.79</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-261.88</td>
</tr>
</tbody>
</table>

... so ROR is 10%
At the end, the unpaid balance is zero.

Figure 7.1

Chapter 7 - Rate of Return Analysis: Single Alternative

- Definition and Concept
  - Rate of Return Calculation Using PW/AW Equation
  - Cautions when Using the ROR Method
  - Multiple ROR values
  - Composite ROR

Chapter 7 - Rate of Return Analysis: Single Alternative
Chapter 7 Rate of Return Analysis: Single Alternative

Definition and Concept

"Now consider an investment

Assume you invest $1000 over 4 years
The investment generates $315.47 per year

What interest rate equals the positive cash flows to the initial investment?

Solution:

We can state: 0 = -1000 + 315.47(P/A,i*,4)

and solve for i* to get i*=10%.

So, 10% is the unique rate that will make the unrecovered investment balance to be zero at the end of the life. Then 10% is the ROR.

Example 7.2: Consider the following cash-flow diagram, show that it has an ROR = 10%

$400 $370 $240 $220

$1,000

Example 7.2, Solution:

\[
\begin{array}{c|c|c|c|c|c}
  n & 0 & 1 & 2 & 3 & 4 \\
  \text{Amount owed} & -1,000 & -1,000 & -700 & -400 & -200 \\
  \text{Interest owed (10%)} & 0 & -100 & -70 & -40 & -20 \\
  \text{Amount paid} & 0 & 400 & 370 & 240 & 220 \\
  \text{Unpaid balance} & -1,000 & -700 & -400 & -200 & 0 \\
\end{array}
\]

The above unpaid balance is positive. This is a surplus balance. Therefore, the last payment is larger than what was needed to have the balance equal zero, and then \( i = 0.09 \) is not the ROR.

\[
\text{PW}(9\%) = -1,000 + \frac{400}{1.09} + \frac{370}{1.09^2} + \frac{240}{1.09^3} + \frac{220}{1.09^4} = 19.57
\]

\[
\text{FW}(9\%) = 19.57(1.09)^4 = 27.63
\]

Example 7.3: Calculate the unpaid balance at the end of the 4th year for the cash flow in Example 7.2 if

a) the interest rate = 9%
b) the interest rate = 11%

Example 7.3, Solution:

a) Interest Rate = 9%

\[
\begin{array}{c|c|c|c|c|c}
  n & 0 & 1 & 2 & 3 & 4 \\
  \text{Amount owed} & -1,000 & -1,000 & -690 & -382.10 & -176.48 \\
  \text{Interest owed (9%)} & 0 & -90 & -62.10 & -34.39 & -15.88 \\
  \text{Amount paid} & 0 & 400 & 370 & 240 & 220 \\
  \text{Unpaid balance} & -1,000 & -690 & -382.10 & -176.48 & 27.63 \\
\end{array}
\]

The above unpaid balance is positive. This is a surplus balance. Therefore, the last payment is larger than the income. This means that after the last payment an unpaid balance still exists. This is a slack balance. Again, \( i = 0.11 \) is not the ROR.

\[
\text{NPW}(0.11) = -1,000 + \frac{400}{1.11} + \frac{370}{1.11^2} + \frac{240}{1.11^3} + \frac{220}{1.11^4} = -18.93
\]

\[
\text{FW}(0.11) = -18.93259(1.11)^4 = -28.74
\]

b) Interest Rate = 11%

\[
\begin{array}{c|c|c|c|c|c}
  n & 0 & 1 & 2 & 3 & 4 \\
  \text{Amount owed} & -1,000 & -1,000 & -710 & -418.10 & -224.09 \\
  \text{Interest owed (11%)} & 0 & -110 & -78.10 & -45.99 & -24.65 \\
  \text{Amount paid} & 0 & 400 & 370 & 240 & 220 \\
  \text{Unpaid balance} & -1,000 & -710 & -418.1 & -224.09 & -28.74 \\
\end{array}
\]

The above unpaid balance is negative. Thus, the debt is larger than the income. This means that after the last payment an unpaid balance still exists. This is a slack balance. Again, \( i = 0.11 \) is not the ROR.

\[
\text{NPW}(0.11) = -1,000 + 400(1.11) + 370(1.11^2) + 240(1.11^3) + 220(1.11^4) = -18.93
\]

\[
\text{FW}(0.11) = -18.93259(1.11)^4 = -28.74
\]
Example 7.3, Solution:

Rate of Return Calculation Using PW/AW Equation

- We can find ROR = i* by either relations:
  - PW:
    \[ PW_0 + PW_r = 0 \]
    Where:
    - \( PW_0 \) - present worth of all cash disbursements
    - \( PW_r \) - present worth of all cash receipts
  - AW:
    \[ AW_0 + AW_r = 0 \]
    Where:
    - \( AW_0 \) - annual worth of all cash disbursements
    - \( AW_r \) - annual worth of all cash receipts

- Once the equation is set up, solve for \( i^* \) (root of the ROR equation) - found by trial and error, numerical methods, MATLAB or Excel.
- If \( i^* \geq \text{MARR} \), accept, alternative is economically viable.
- If \( i^* < \text{MARR} \), the alternative is not economically viable.
- Here, we assume that PW as a function of \( i \) “looks like” the fig. 7.3 (slide 13)
- Strictly speaking, for ROR analysis to be consistent with PW analysis at MARR, we need to have:
  \[ \text{PW}(i) \geq 0 \text{ for } i \leq i^* \text{ and negative otherwise.} \]

Example 7.4: Consider the following cash flow and compute the ROR using a present worth equation.

Year | 0  | 1  | 2  | 3  | 4  | 5  |
-----|----|----|----|----|----|----|
Cash Flow | -1,000 | 0  | 0  | +500 | 0  | +1,500 |

Solution:

\[ 500(P/A,i\%,3) + 1,500(P/F,i\%,5) – 1,000 = 0 \]

By trial and error:

- \( i = 16\%: 500(0.6407) + 1,500(0.4761) – 1,000 = 34.5 > 0 \)
- \( i = 18\%: 500(0.6086) + 1,500(0.4371) – 1,000 = -40.05 < 0 \)

by interpolation \( i = 16.9\% \) (Verify that the errors is < $1)

Example 7.5:

Consider the following cash flow and compute the ROR using a present worth equation.

Year | 0  | 1-10 | 10 |
-----|----|------|----|
Cash Flow | -5,000 | $100 | $7,000 |

Solution:

\[ 100(P/A,i\%,10) + 7,000(P/F,i\%,10) – 5,000 = 0 \]
Example 7.5. Solution (continue):

i=5%: 
100(7.7217) + 7,000(0.6139) – 5,000 = 69.47 >0 
\[ i > 5\% \]

i=6%:
100(7.3601) + 7,000(0.5584) – 5,000 = -355.19 <0 

by interpolation: 
\[
\begin{array}{c|c|c|c}
\hline
i & 5 & 6 & 5.16 \\
\hline
\text{AWD} + \text{AWR} & 69.47 & -355.19 & \text{by interpolation, } i = 5.16\% \\
\hline
\end{array}
\]

Example 7.6: Same cash flow as in previous example, compute the ROR using an annual worth equation.

Solution:
\[
\text{AWD} + \text{AWR} = 0
\]

\[
100 + 7,000 \left( \frac{A/F,i\%}{10} \right) – 5,000 \left( \frac{A/P,i\%}{10} \right) = 0
\]

i=5%: 100 + 7,000(0.0795) –5,000(0.12950) = 9 

i=6%: 100 + 7,000(0.07587) –5,000(0.13587) = -48.26

by interpolation, \( i = 5.16\% \)

Rate of Return Calculation Using Excel

- The fastest way to determine an \( i^\ast \) value by computer, when there is a series of equal cash flows (A series), is to apply the RATE function. This is a powerful one-cell function, where it is acceptable to have a separate \( P \) value in year 0 and an \( F \) value in year \( n \). The format is

\[
\text{RATE}(n,A,P,F)
\]

(The \( F \) value does not include the series \( A \) amount)

Rate of Return Calculation Using Excel

- When cash flows vary from period to period, the best way to find \( i^\ast \) is to enter the net cash flows into contiguous cells (including any $0 amounts) and apply the IRR function in any cell. The format is

\[
\text{IRR(first_cell:last_cell,guess)}
\]

where "guess" is the \( i \) value at which the computer starts searching for \( i^\ast \).

Cautions when Using the ROR Method

- Could have multiple \( i^\ast \) values.
- Special procedure for multiple alternatives is needed.
- ROR can take values from \(-100\%\) through infinite.

Multiple ROR values

- A class of ROR problems exist that will possess multiple \( i^\ast \) values.
- Two tests can be applied:
  - Descartes’ rule of signs
  - Norstrom’s cumulative cash flow sign test
Multiple ROR values

- **Conventional cash flow series** (also called simple cash flow series): Algebraic signs on the net cash flows changes only once

  **Example**
  
<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow 1</th>
<th>Cash Flow 2</th>
<th>Cash Flow 3</th>
<th>Cash Flow 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>1</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

  In all the cash flow examples shown above, the sign changes either from – to + or vice versa.

- **Non-conventional cash flow series** (also called non-simple cash flow series): More than one sign change on the net cash flow. In this case, it's possible that there will be multiple i* values.

  **Example**
  
<table>
<thead>
<tr>
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  In all the cash flow examples shown above, the sign changes either from – to + or vice versa.

Norstrom’s cumulative cash flow sign test:

- Let \( \{A_0, A_1, \ldots, A_n\} \) be net cash flows and let \( \{C_0, C_1, \ldots, C_n\} \) be cumulative cash flows: \( C_i = \sum_{t=0}^{i} A_t \)

  A cash flow \( \{A_0, A_1, \ldots, A_n\} \) with cumulative cash flow \( \{C_0, C_1, \ldots, C_n\} \) will have a unique non-negative internal rate of return if the cumulative cash flow changes signs once and \( C_n \neq 0 \).

  **Note**: -0,0,…,0,+ (or) +,0,0,…,- is ONE sign change.

Norstrom’s Criterion

- It is not necessary for the cash flow to start negative as stated in the book. Only the number of sign changes in the cash flow is important.

- It is only a **sufficient** condition, not necessary. Text book says (p250) “Only if \( S_0 < 0 \) and signs change one time in the series is there a single, real-number, positive i*” – this is incorrect.
  
  - If it is satisfied THEN there exists a unique positive root.
  - If it is not satisfied … there still may be a unique positive root.
  - Net cash flow of \( -1,2,-2,4 \) has a unique i* = 1, although the cumulative cash flow \( -1,1,-1,3 \) changes sign three times.

## Multiple ROR values

- **Conventional cash flow series** (also called simple cash flow series): Algebraic signs on the net cash flows changes only once

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<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>1</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

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<tbody>
<tr>
<td>0</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>1</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>-</td>
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</tr>
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  **Note**: -0,0,…,0,+ (or) +,0,0,…,- is ONE sign change.

Norstrom’s Criterion

- It is not necessary for the cash flow to start negative as stated in the book. Only the number of sign changes in the cash flow is important.

- It is only a **sufficient** condition, not necessary. Text book says (p250) “Only if \( S_0 < 0 \) and signs change one time in the series is there a single, real-number, positive i*” – this is incorrect.
  
  - If it is satisfied THEN there exists a unique positive root.
  - If it is not satisfied … there still may be a unique positive root.
  - Net cash flow of \( -1,2,-2,4 \) has a unique i* = 1, although the cumulative cash flow \( -1,1,-1,3 \) changes sign three times.
Example 7.8

**Multiple ROR values**

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
<th>Cumulative</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2000</td>
<td>-2000</td>
</tr>
<tr>
<td>2</td>
<td>-2000</td>
<td>-4000</td>
</tr>
<tr>
<td>3</td>
<td>+2500</td>
<td>-1500</td>
</tr>
<tr>
<td>4</td>
<td>-500</td>
<td>-2000</td>
</tr>
<tr>
<td>5</td>
<td>+600</td>
<td>-1400</td>
</tr>
<tr>
<td>6</td>
<td>+500</td>
<td>-900</td>
</tr>
<tr>
<td>7</td>
<td>+400</td>
<td>-500</td>
</tr>
<tr>
<td>8</td>
<td>+300</td>
<td>-200</td>
</tr>
<tr>
<td>9</td>
<td>+200</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>+100</td>
<td>+100</td>
</tr>
</tbody>
</table>

Descartes’ test: up to three RORs.

Norstrom’s test: one unique positive root.

Example 7.9: Consider the following cash flows and compute the ROR using the present worth equation.

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
<th>Cumulative Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+2000</td>
<td>-$2000</td>
</tr>
<tr>
<td>1</td>
<td>-500</td>
<td>-$1500</td>
</tr>
<tr>
<td>2</td>
<td>-8100</td>
<td>-$9000</td>
</tr>
<tr>
<td>3</td>
<td>+6800</td>
<td>+200</td>
</tr>
</tbody>
</table>

Solution:

\[
PW = 2000 + 6800(P/F,i\%\%,3) - 500(P/F,i\%\%,1) - 8100(P/F,i\%\%,2)
\]

There are 2 sign changes, so there could be at most 2 i* values.

Example 7.9, solution (continue):

Solved by Excel, i* = 7.47% or 41.35%

Example 7.10: Consider the following cash flow and identify the number of ROR values that could satisfy the present worth equation.

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
<th>Cumulative Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$2000</td>
<td>-$2000</td>
</tr>
<tr>
<td>1</td>
<td>+500</td>
<td>-$1500</td>
</tr>
<tr>
<td>2</td>
<td>-200</td>
<td>-$1700</td>
</tr>
<tr>
<td>3</td>
<td>+3000</td>
<td>+$1300</td>
</tr>
</tbody>
</table>

Solution:

There are 3 sign changes. So by Descartes’ Rule, there are at most 3 ROR values that satisfy the present worth equation. However, by Norstrom’s criterion, this cash flow has a unique ROR: 20.31% (by Excel.)

Example 7.10, Solution (Continue):

Note: In this case, PW is monotone decreasing in i.
Chapter 7 Rate of Return Analysis: Single Alternative

**Composite ROR**

- **ROR** is the rate of interest earned on the unrecovered balance of an investment.
- When some positive net cash flow occurs, these are not considered further in an internal rate of return calculation.
- Project net-investment procedure: any net positive cash flow is assumed to be reinvested at reinvestment rate, c.

**Composite Rate of Return** $i'$ is the unique rate of return for a project which assumes that net positive cash flow which represents money not immediately needed by the project are reinvested at the reinvestment rate c.

- Relationship between c, $i^*$ and $i'$
  1. if $c = i^*$ then $i' = i^*$
  2. if $c < i^*$ then $i' < i^*$
  3. if $c > i^*$ then $i' > i^*$

**Example 7.11:**

Consider the following cash flow and determine the ROR. Assume a reinvestment rate c of 15% and determine the composite rate of return.

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
<th>Cumulative Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+$50</td>
<td>+$50</td>
</tr>
<tr>
<td>1</td>
<td>-$200</td>
<td>-$150</td>
</tr>
<tr>
<td>2</td>
<td>+$50</td>
<td>-$100</td>
</tr>
<tr>
<td>3</td>
<td>+$100</td>
<td>0</td>
</tr>
</tbody>
</table>

**Solution:**

1) Unconventional cash flow as there are 2 sign changes.

2) Final cumulative is 0 so we cannot apply Norstrom’s criterion to check the existence of a unique ROR. One solution is $i^*$=0; what is the other one?

**Solution (continue):**

From Fig. 7.5, it is clear that at $i$=0% and $i$=275%, the PW is 0. But which should we choose?

![Figure 7.5 PW vs. Interest Rates (Example 7.11)](image-url)
Solution (continue):

In order to overcome this confusion, we can solve the problem using the Project Net-Investment Procedure.

\[ F_0 = +\$50 > 0 \text{ so we can invest this at } c=15\% \]
\[ F_1 = F_0 (1+c) -200 = 50(1.15)-200 = -$142.50 < 0 \]
\[ F_2 = F_1(1+i') + 50 = -142.50(1+i') + 50 < 0, \text{ for all } i'>0 \]
\[ F_3 = F_2 (1+i') + 100 = -142.5(1+i')(1+i') + 50(1+i') + 100 = 0 \]

Solving for \( i' \) gives 3.13%

Example 7.12: Consider the following cash flow.
Assume the reinvestment rate as 15% per year and determine the composite rate of return.

<table>
<thead>
<tr>
<th>Yr</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>NCF</td>
<td>-200</td>
<td>100</td>
<td>50</td>
<td>-1,800</td>
<td>600</td>
<td>500</td>
<td>400</td>
<td>300</td>
<td>200</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>CCF</td>
<td>-200</td>
<td>300</td>
<td>350</td>
<td>-1,450</td>
<td>-850</td>
<td>-350</td>
<td>50</td>
<td>350</td>
<td>550</td>
<td>650</td>
<td></td>
</tr>
</tbody>
</table>

Example 7.12, Solution:

Unconventional cash flow as there are 2 sign changes.

More than one sign change in the cumulative cash flows so there may be more than one positive real root.

Computing the present worth for different \( i^* \) values, as shown below, would help us to find the two \( i^* \) values as 29% and 48%. But which should we choose?

<table>
<thead>
<tr>
<th>( i^* )</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>25%</th>
<th>30%</th>
<th>35%</th>
<th>40%</th>
<th>45%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present Worth</td>
<td>$364.4</td>
<td>$196.0</td>
<td>$97.7</td>
<td>$41.9</td>
<td>$11.8</td>
<td>$(2.6)</td>
<td>$(7.6)</td>
<td>$(7.1)</td>
<td>$(5.4)</td>
<td>$(2.0)</td>
</tr>
</tbody>
</table>

Example 7.12, Solution:
Solve the problem by Project Net-Investment Procedure.

\[ F_0 = 0 \]
\[ F_1 = 0 + 200 = 200 > 0; \text{ so use } c. \]
\[ F_2 = 200(1.15) + 100 = 330 > 0; \text{ so use } c. \]
\[ F_3 = 330(1.15) + 50 = 429.50 > 0; \text{ so use } c. \]
\[ F_4 = 429.50 (1.15) -1,800 = -1,306.08 < 0; \text{ so use } i'. \]
\[ F_5 = -1,306.08(1+i') + 600; < 0 \text{ for all } i'>0, \text{ in this case, use } i' \]
\[ F_5 = F_4(1+i') + 500 \]
\[ F_6 = F_5(1+i') + 400 \]
\[ F_7 = F_6(1+i') + 300 \]
\[ F_8 = F_7(1+i') + 200 \]
\[ F_9 = F_8(1+i') + 100 \]
\[ F_{10} = 0 \Rightarrow i' = 21.25\% \]

Note: \( c = 15\% < i_1 = 29\% \) and \( i_2 = 48\% \) as a result \( i' < i. \)