Chapter 6 – Annual Worth Analysis

### Advantages and Uses

- Ideal approach for comparing alternatives with different lives under LCM assumptions
- AW value has to be calculated for only one life cycle
- LCM comparison is implicit as, \( AW_{LCM} = AW_{Life} \)
- Popular and easily understood
- Results are reported in $/time period

### Capital Recovery and AW Value

- **Capital Recovery** is the equivalent annual cost of obtaining the asset plus the salvage
- CR is a function of \( \{ P, SV, i\%, \text{ and } n \} \)
- AW is comprised of two components: capital recovery for the initial investment \( P \) at a stated interest rate (MARR) and the equivalent annual amount \( A \)

### An alternative usually has the following cash flow estimates:

- **Initial Investment** \( (P) \) – the total first cost of all assets and services required to initiate the alternative.
- **Salvage Value** \( (SV) \) – the terminal estimated value of assets at the end of their useful life.
- **Annual Amount** \( (A) \) – the equivalent annual amount; typically this is the annual operating cost (AOC).

### Assume \( P, SV \) and \( A \) are just the magnitudes, to find CR:

- **Method I** : Compute AW of the original cost and add the AW of the salvage value
  \[
  CR = - P(A|P, i, n) + SV(A|F, i, n)
  \]
- **Method II** : Add the present worth of the salvage value to the original cost, then compute the annual worth of the sum.
  \[
  CR = [- P + SV(P|F, i, n)] (A|P, i, n)
  \]

\[
AW = CR - A \quad \text{(Note the difference from the book)}
\]
Example 6.1: A contractor purchased a used crane for $11,000. His operating cost will be $2700 per year, and he expects to sell it for $5000 five years from now. What is the equivalent annual worth of the crane at an interest rate of 10%?

Solution:
\[
CR = -11,000(A/P, 10\%, 5) + 5000(A/F, 10\%, 5)
\]
\[
AW = -11,000(A/P, 10\%, 5) + 5000(A/F, 10\%, 5) - 2700
\]
\[
= -11,000(0.2638) + 5000(0.1638) - 2700
\]
\[
= -$4782.8
\]

Example 6.2: Calculate the AW for the following cash flow. Assume the MARR is 12% per year

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8 million</td>
</tr>
<tr>
<td>1</td>
<td>5 million</td>
</tr>
<tr>
<td>1-8</td>
<td>0.9 million</td>
</tr>
<tr>
<td>8</td>
<td>0.5 million</td>
</tr>
</tbody>
</table>

First find the capital recovery (CR)

Method I:
\[
CR = [-8.0 - 5.0(P/F,12\%,1)](A/P,12\%,8) + 0.5(A/F,12\%,8)
\]
\[
= [-8.0-5.0*(.8929)](0.2013) + 0.5*(0.0813)
\]
\[
= -$2.47 million
\]

Method II:
\[
CR = [-8.0 - 5.0(P/F,12\%,1) + 0.5(P/F,12\%,8)](A/P,12\%,8)
\]
\[
= [-8.0-5.0*(.8929) + 0.5*(.4039)](0.2013)
\]
\[
= -$2.47 million
\]

\[
AW = CR - A
\]
\[
= -2.47 - 0.9 = -$3.37 million
\]

Evaluating Alternatives by AW Analysis

- For mutually exclusive alternatives, calculate AW over one life cycle at the MARR
- One alternative: AW ≥ 0, MARR is met or exceeded
- Two or more alternatives: Choose the alternative with numerically largest AW value
- Note that you are making a comparison over LCM to ensure equal service
- Your calculations are simplified since AW over LCM is the same as AW over life cycle

Example 6.3: The following costs are estimated for two equal-service tomato-peeling machines to be evaluated by a canning plant manager.

<table>
<thead>
<tr>
<th></th>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Cost, $</td>
<td>26,000</td>
<td>36,000</td>
</tr>
<tr>
<td>Annual maintenance cost, $</td>
<td>800</td>
<td>300</td>
</tr>
<tr>
<td>Annual labor cost, $</td>
<td>11,000</td>
<td>7,000</td>
</tr>
<tr>
<td>Extra annual income taxes, $</td>
<td>-</td>
<td>2,600</td>
</tr>
<tr>
<td>Salvage value, $</td>
<td>2,000</td>
<td>3,000</td>
</tr>
<tr>
<td>Life, years</td>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>

If the minimum required rate of return is 15% per year, help the manager decide which machine to select.
### Solution:

**Machine A:**

\[ AW_A = -26,000(A/P, 15\%, 6) + 2,000(A/F, 15\%, 6) - 11,800 \]
\[ = -26,000(0.26424) + 2,000(0.11424) - 11,800 \]
\[ = -18,442 \]

**Machine B:**

\[ AW_B = -36,000(A/P, 15\%, 10) + 3,000(A/F, 15\%, 10) - 9,900 \]
\[ = -36,000(0.19925) + 3,000(0.04925) - 9,900 \]
\[ = -16,925 \]

Select machine B since \( AW_B > AW_A \).

### Example 6.4:

Assume the company in previous example is planning to exit the tomato canning business in 4 years. At that time, the company expects to sell machine A for $12,000 or machine B for $15,000. All other costs are expected to remain the same. Which machine should the company purchase under these conditions?

**NOTE:**

This is a study period problem. So we have considered all cash flows only for the study period (4 years).

**Solution:**

\[ AW_A = -26,000(A/P, 15\%, 4) + 12,000(A/F, 15\%, 4) - 11,800 \]
\[ = -26,000(0.35027) + 12,000(0.20027) - 11,800 \]
\[ = -18,504 \]

\[ AW_B = -36,000(A/P, 15\%, 4) + 15,000(A/F, 15\%, 4) - 9,900 \]
\[ = -36,000(0.35027) + 15,000(0.20027) - 9,900 \]
\[ = -19,506 \]

Select machine A as \( AW_A > AW_B \).

### Example 6.5:

A public utility is trying to decide between two different sizes of pipe for a new water main. A 250-mm line will have an initial cost of $40,000, whereas a 300-mm line will cost $46,000. Since there is more head loss through the 250-mm pipe, the pumping cost for the smaller line is expected to be $2,500 per year more than for the 300-mm line. If the pipes are expected to last for 15 years, which size should be selected if the interest rate is 12% per year? Use an annual-worth analysis.

**Solution:**

\[ AW_{250} = -40,000(A/P, 12\%, 15) - 2,500 \]
\[ = -8,373 \]

\[ AW_{300} = -46,000(A/P, 12\%, 15) \]
\[ = -6,754 \]

Select the 300 mm pipe.

### Reminder: Capitalized Cost (CC)

**Capitalized Cost (CC) for a uniform series A of end-of-period cash flows:**

\[ P = A(P/A, i, n) = A[1/(1+i)^n] - 1/[i(1+i)^n] \]
\[ = A \left[ 1 - \frac{1}{(1+i)^n} \right] \]
\[ = A / i \]

Now, we have: \( CC = A/i \)

Also,

\[ A = CC(i) \]
### Annual-Worth of a Permanent Investment

If an investment has infinite life, it is called a **perpetual (permanent) investment**. If $P$ is the present worth of the cost of that investment, then $AW$ is $P$ times $i$.

$$AW = P \cdot i$$

### Example 6.6:

Two alternatives are considered for covering a football field. The first is to plant natural grass and the second is to install AstroTurf. Interest rate is 10%. Cost structure for each alternative is given below.

### Alternative I:

Natural Grass - Replanting will be required each 10 years at a cost of $10,000. Annual cost for maintenance is $5,000. Equipment must be purchased for $50,000 which will be replaced after 5 years with a salvage value of $5,000.

### Alternative II:

AstroTurf - Installing AstroTurf cost $150,000 and it is expected to last indefinitely. Annual maintenance cost is expected to be $5,000.

### Solution:

**AW of alt. A** 

**Cycle = 10 years**

- **Planting**: $-10,000 (A|P, .10, 10) = -$1,628
- **1st Set Equipment (first 5 years)**: $\{-50,000 + 5,000(P|F, .10, 5)\} (A|P, .10, 10) = -$7,632
- **2nd Set of Equipment (second 5 years)**: $\{\{[-50,000 + 5,000(P|F, .10, 5)] (P|F, .10, 5)\} (A|P, .10, 10) = -$4,739

**AW of Alternative A, continued**

- **Maintenance**: $-5,000 annually
- **Total**: $-1,628 - 7,632 - 4,739 - 5,000 = -$18,999
AW of Alternative B:

(AstroTurf - Installing AstroTurf cost $150,000 and it is expected to last indefinitely. Annual maintenance cost is expected to be $5,000)

Annual Cost of Installation: $-150,000 (.10) = $-15,000
Maintenance: $-5,000 annually
Total: $-20,000

Choose A

Example 6.7: Compare the following proposals to maintain a canal. Use interest rate 5%.

Proposal A (Buying Dredging Machine)
First Cost, $ 65,000
Annual maintenance cost, $ 32,000
Salvage value, $ 7,000
Life, years 10

Proposal B (Concrete Lining)
Initial cost, $ 650,000
Annual maintenance cost, $ 1,000
Lining repairs every 5 years, $ 1,800
Life, years permanent

Solution:

\[ AW_A = -65,000(A/P,5\%,10)+7,000(A/F,5\%,10) = 39,861 \]
\[ AW_B = -650,000(0.05) - 1,000 - 1,800(A/F,5\%,5) = -33,826 \]

Choose proposal B.

Example 6.16

The cash flow associated with a project having an infinite life is $-100,000 now, $-30,000 each year, and an additional $-50,000 every 5 years beginning 5 years from now. Determine its perpetual equivalent annual worth at an interest rate of 20% per year.

Solution

\[ AW = -100,000(0.20) - 30,000 - 50,000(A/F,20\%,5) = -56,719 \text{ per year} \]

Example 6.17

A philanthropist working to set up a permanent endowment wants to deposit a uniform amount of money each year, starting now and for 10 more (11 deposits), so that $10 million per year will be available for research related to planetary colonization. If the first $10 million grant is to be awarded 11 years from now, what is the size of the uniform donations, if the fund will generate income at a rate of 15% per year?
Solution

First find P in year 10 for the $10 million annual amounts and then use the A/F factor to find A:

\[ P_{10} = \frac{-10}{0.15} = -66.667 \text{ million} \]

\[ A = -66.667 \times (A/F, 15\%, 11) = -2,738,000 \text{ per deposit} \]

Example 6.18

- The costs associated with a certain robotic arm are $40,000 now and $24,000 per year, with a $6000 salvage value after 3 years. Determine the perpetual equivalent annual worth of the robot at an interest rate of 20% per year.

Solution

The perpetual uniform annual worth is the AW for one life cycle:

\[ AW = -40,000 \times (A/P, 20\%, 3) - 24,000 + 6000 \times (A/F, 20\%, 3) = -41,341 \]