Chapter 5 – Present Worth Analysis

Purpose

- To compare mutually exclusive alternatives based on present worth, under the assumption that each alternative is expected to provide the same service.
- Generally, the cash flows to be considered are: first cost, annual costs, non-recurring costs, revenues, and salvage value.
- The cash flow series can be finite or infinite.

Different Types of Projects

- Mutually exclusive: Only one of the viable projects can be selected by the economic analysis.
- Independent: More than one viable project may be selected by the economic analysis.
- Do-nothing (DN)

Present Worth of Single Alternative

1. Compute the present worth (PW) at MARR;
2. If \( PW \geq 0 \), the alternative is financially viable;
3. Otherwise, the alternative is not financially viable.

Present Worth of Equal-life Alternatives

1. Calculate the PW of each alternative at the MARR;
2. Select the alternative with the PW value that is numerically largest (i.e., least negative or most positive).
Present Worth of Equal-life Alternatives

Example 5.1:

<table>
<thead>
<tr>
<th></th>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Cost</td>
<td>$2,500</td>
<td>$3,500</td>
</tr>
<tr>
<td>Annual Operating Cost</td>
<td>900</td>
<td>700</td>
</tr>
<tr>
<td>Salvage Value</td>
<td>200</td>
<td>350</td>
</tr>
<tr>
<td>Life</td>
<td>5 years</td>
<td>5 years</td>
</tr>
</tbody>
</table>

\( i = 10\% \) per year

Example 5.1, Solution:

\[
PA = -2,500 - 900 \cdot (P|A, .10, 5) + 200 \cdot (P|F, .10, 5) \\
= -2,500 - 900 \cdot (3.7908) + 200 \cdot (0.6209) \\
= -2,500 - 3,411.72 + 124.18 = -5,788
\]

\[
PB = -3,500 - 700 \cdot (P|A, .10, 5) + 350 \cdot (P|F, .10, 5) \\
= -3,500 - 2,653.56 + 217.31 = -5,936
\]

\( PA > PB \) SELECT MACHINE A

Present Worth of Equal-life Alternatives

Example 5.2:

XYZ Car rental company is considering purchasing one type of compact car to replenish its fleet. Two companies are offering this kind of car. Which option should XYZ choose if their MARR is 10% per year?

<table>
<thead>
<tr>
<th></th>
<th>Company A</th>
<th>Company B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchase cost</td>
<td>$12,000</td>
<td>$14,000</td>
</tr>
<tr>
<td>Maintenance Cost</td>
<td>$900</td>
<td>$500</td>
</tr>
<tr>
<td>Salvage value</td>
<td>$4,000</td>
<td>$5,000</td>
</tr>
<tr>
<td>Life (Years)</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Example 5.2, Solution:

\[
PWA = -12,000 - 900 \cdot (P|A, 10\%, 3) + 4,000 \cdot (P|F, 10\%, 3) \\
= -12,000 - 900 \cdot 2.4869 + 4,000 \cdot 0.7513 \\
= -11,233.01
\]

\[
PWB = -14,000 - 500 \cdot (P|A, 10\%, 3) + 5,000 \cdot (P|F, 10\%, 3) \\
= -14,000 - 500 \cdot 2.4869 + 5,000 \cdot 0.7513 \\
= -11,486.95
\]

\( PWB < PWA \), XYZ should choose car made by Company A.

Present Worth of Different-life Alternatives

Two methods in different-life alternatives analysis:

1. Compare the alternatives over a period of time equal to the least common multiple (LCM) of their lives.

Assumptions:

a. The service provided by the alternatives under consideration will be needed for LCM years or more.

b. The cash flow estimates of the alternatives will be the same in all subsequent life cycles as they were in the first one.
Present Worth of Different-life Alternatives

2. Compare the alternatives using a study period of length n years, which does not necessarily take into consideration the useful lives of the alternatives.

Note:
- The study period method is used when the first assumption about the length of time the alternatives will be needed cannot be made.
- Or, it is also useful when the LCM method yields an unrealistic evaluation period.

EXAMPLE 5.3:

<table>
<thead>
<tr>
<th>Machine</th>
<th>First Cost</th>
<th>Annual Operating Cost</th>
<th>Salvage Value</th>
<th>Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$11,000</td>
<td>3,500</td>
<td>1,000</td>
<td>6</td>
</tr>
<tr>
<td>B</td>
<td>$18,000</td>
<td>3,100</td>
<td>2,000</td>
<td>9</td>
</tr>
</tbody>
</table>

\[ i = 15\% \text{ per year} \]

LCM(6, 9) = 18 year study period will apply for present worth

Machine A

Cycle 1 for A

Cycle 2 for A

Cycle 3 for A

6 years

6 years

6 years

Machine B

Cycle 1 for B

Cycle 2 for B

9 years

9 years

18 years

LCM = 18 years

Calculate the present worth of a 6-year cycle for alternative A

\[ PW_A = -11,000 - 3,500 \left( P | A, .15, 6 \right) + 1,000 \left( P | F, .15, 6 \right) \]

\[ = -11,000 - 3,500 \left( 3.7845 \right) + 1,000 \left( .4323 \right) \]

\[ = -23,813 \text{ which occurs at time 0, 6 and 12} \]

Now, for the 18 years:

\[ PW_A = -23,813 - 23,813 \left( P | F, .15, 6 \right) - 23,813 \left( P | F, .15, 12 \right) \]

\[ = -23,813 \times 0.4323 = -38,558 \]

Machine B

Cycle 1 for B

Cycle 2 for B

9 years

9 years

18 years

LCM = 18 years

Calculate the present worth of a 9-year cycle for alternative B

\[ PW_B = -18,000 - 3,100 \left( P | A, .15, 9 \right) + 2,000 \left( P | F, .15, 9 \right) \]

\[ = -18,000 - 3,100 \left( 4.7716 \right) + 2,000 \left( 0.2843 \right) \]

\[ = -32,223 \text{ which occurs at time 0 and 9} \]

Now, for the 18 years:

\[ PW_B = -32,223 - 32,223 \left( P | F, .15, 9 \right) \]

\[ = -32,223 \times 0.2843 = -41,384 \]

\[ PW_A > PW_B, \text{ so the company should choose machine A.} \]
Present Worth of Different-life Alternatives

Guidelines on Study period method:
1. Only consider cash flows during the study period; ignore those beyond
2. An estimated market value at the end of the study period must be made

Example 5.4:
Continue with previous example, which machine should the company choose if the interest rate is 15% per year, and the study period is 5 years? Assume that the salvage values at the end of the study period are the same as the previous case.

Machine A:
PWA = -11,000 – 3,500(P/A,15%,5) + 1,000(P/F,15%,5)
= -11,000 – 3,500(3.3522) + 1,000(0.4972)
= $- 22,235.50

Machine B:
PWB = -18,000 – 3,100(P/A,15%,5) + 2,000(P/F,15%,5)
= -18,000 – 3,100(3.3522) + 2,000(0.4972)
= $- 27,397.42

Decision:
PWA > PWB, so the company should choose machine A.

Future Worth Analysis

The future worth (FW) may be determined
1. Directly from the cash flows by determining the future worth value (using LCM or study period), or
2. By multiplying the PW value by the F/P factor at the established MARR (value of n depends upon which time period has been used to determine PW – LCM or study period).

Applications: Projects that do not come on-line until the end of the investment period, like
- Toll Roads
- Electric generation facilities
- Hotels
- Commercial Buildings
Capitalized Cost (CC) – Permanent Alternatives

- Capitalized Cost (CC): the present worth of an alternative that will last “forever”. Some examples are:
  - Public sector projects
  - Bridges
  - Dams
  - Irrigation Systems
  - Railroads

Capitalized Cost (CC)

- Capitalized Cost (CC) for a uniform series A of end-of-period cash flows:
  \[
P = A \frac{(1+i)^n - 1}{i(1+i)^n}
\]

  \[\lim_{n \to \infty} A \frac{1 - \frac{1}{(1+i)^n}}{i} = \frac{A}{i}\]

  Now, we have:  \( CC = \frac{A}{i} \)
  Also, \( A = CC(i) \)

Example 5.5:

Find the capitalized cost of an infinite series of annual payments equal to $10,000 using an interest rate of 8% compounded annually.

Solution: \( A = 10,000 \) and \( i = 0.08 \).

\[ CC = \frac{10,000}{0.08} = $125,000. \]

Note: \( 10,000 (P/A, 8\%, 100) = $124,943 \) this value is approaching the limit of $125,000.

Example 5.6:

Calculate the Capitalized Cost of a project which has an initial cost of $150,000. The annual operating cost is $8000 for the first 4 years and $5000 thereafter. There is an recurring $15000 maintenance cost every 15 years. Interest is 15% per year.

1. Consider $3000 of the $8000 cost for the first four years to be a one time cost leaving $5000 annual operating cost forever.

\[
P_0 = -150,000 - 3000 (P/A, .15, 4) = -158,565
\]

2. Recurring annual cost is $5000 plus the equivalent annual of the 15,000 end of cycle cost.

\[
A = -5000 - 15,000 (A/F, .15, 15) = 5315
\]

3. Capitalized Cost = \( -158,565 + (-5315/15) = -$193.998 \)
**Capitalized Cost (CC)**

Example 5.7:

Calculate the capitalized cost of a project that has an initial cost of $150,000 and an additional investment cost of $50,000 after 10 years. The annual operating cost will be $5000 for the first 4 years and $8000 thereafter. In addition, there is expected to be a recurring major rework cost of $15,000 every 13 years. Assume that \( i = 15\% \) per year.

**Step 1:**

**Non-recurring costs**

Initial cost $150,000, additional investment cost of $50,000 after 10 years, and ‘+3,000’ adjustment for the first four years to make \( A = -8K \)

\[
CC_1 = -150,000 - 50,000(P/F,15\%,10) + 3,000(P/A,15\%,4) = $-153,795
\]

**Step 2:**

**Recurring costs**

Major rework cost of $15,000 every 13 years

\[
A_1 = -15,000 (A/F,15\%,13) = -15,000\times 0.02911 = $-437
\]

Annual operating cost

\[
A_2 = -8,000
\]

\[
A_1 + A_2 = -$437 - $8,000 = -$8,437
\]

**Step 3:**

Obtain CC value of recurring costs

\[
CC_2 = \frac{-8,437}{0.15} = $-56,247
\]

**Step 4:**

Obtain CC value of recurring costs

\[
CC_2 = \frac{-8,437}{0.15} = $-56,247
\]

**Step 5:**

Capitalized cost

\[
CC = CC_1 + CC_2 = $-210,042
\]

**Shifted Capitalized Costs**

Example 5.8

Determine the capitalized cost of $100,000 at time 0, $25,000 in years 1 through 5, and $50,000 per year from year 6 on. Use an interest rate of 10% per year.

**Solution:**

\[
PW_{cap} = -100,000 - 25,000 (P/A, 10\%, 5) - (50,000 /0.10) \times (P/F, 10\%, 5) = $-505,220.
\]
Capitalized cost analysis for a finite-life alternative

If a finite-life alternative is compared to one with an infinite life on the basis of their capitalized costs, proceed as follows:

To determine capitalized cost for the alternative with a finite life, calculate the equivalent A value for one life cycle and divide by the interest rate.

Example 5.9
Determine the capitalized cost of an alternative that has a first cost of $32,000, an annual maintenance cost of $6000, and a salvage value of $8000 after its 4-year life. Use an interest rate of 14% per year.

Solution:
Find AW through one life cycle, then divide by i:

\[ AW = -32,000 \left(\frac{A}{P}, 14\%, 4\right) - 6000 + 8000 \left(\frac{A}{F}, 14\%, 4\right) = -32,000(0.34320) - 6000 + 8000(0.20320) = -$15,356.80 \]

\[ CC = \frac{-15,356.80}{0.14} = -$109,691 \]

See Ex 5.6 p184 in textbook

Payback Period Analysis

Two forms:
- \( i > 0\% \) (called discounted)
- \( i = 0\% \) (called no-return or simple)

The payback period, \( n_p \), is the estimated time to recover the initial investment \( P \) at a stated rate of return.

- Use as an initial screening or supplemental technique.
- Avoid using this method as the primary measure of worth to select an alternative.
Example 5.10: A window frame manufacturer is searching for ways to improve revenue from its triple-insulated sliding windows. Alternative A is an increase in TV and radio marketing. A total of $600,000 spent now is expected to increase revenue by $100,000 per year. Alternative B requires the same amount for enhancements to the in-plant manufacturing process that will improve the product. New revenues start slowly for this alternative at an estimated $15,000 per year, with a growth of 20% per year as the improved product gains reputation among builders. The MARR is 6%, and maximum projection periods are 8 years for A and 16 years for B. Use payback analysis to select the more economical alternative.

### Payback Period Analysis

**Alternative A:**

<table>
<thead>
<tr>
<th>Time (t)</th>
<th>CF(t)</th>
<th>(P/F, 6%, t)</th>
<th>PW</th>
<th>Accum PW</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$600,000</td>
<td>1.00000</td>
<td>-$600,000</td>
<td>-$600,000</td>
</tr>
<tr>
<td>1</td>
<td>$100,000</td>
<td>0.94340</td>
<td>$94,340</td>
<td>-$505,660</td>
</tr>
<tr>
<td>2</td>
<td>$100,000</td>
<td>0.89000</td>
<td>$89,000</td>
<td>-$416,661</td>
</tr>
<tr>
<td>3</td>
<td>$100,000</td>
<td>0.83962</td>
<td>$83,962</td>
<td>-$332,699</td>
</tr>
<tr>
<td>4</td>
<td>$100,000</td>
<td>0.79209</td>
<td>$79,209</td>
<td>-$253,489</td>
</tr>
<tr>
<td>5</td>
<td>$100,000</td>
<td>0.74726</td>
<td>$74,726</td>
<td>-$178,764</td>
</tr>
<tr>
<td>6</td>
<td>$100,000</td>
<td>0.70496</td>
<td>$70,496</td>
<td>-$108,268</td>
</tr>
<tr>
<td>7</td>
<td>$100,000</td>
<td>0.66506</td>
<td>$66,506</td>
<td>-$41,762</td>
</tr>
<tr>
<td>8</td>
<td>$100,000</td>
<td>0.62741</td>
<td>$62,741</td>
<td>$20,979</td>
</tr>
</tbody>
</table>

Payback for A is between 7 and 8 years (7.67 years)

**Alternative B:**

<table>
<thead>
<tr>
<th>Time (t)</th>
<th>CF(t)</th>
<th>(P/F, 6%, t)</th>
<th>PW</th>
<th>Accum PW</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$600,000</td>
<td>1.00000</td>
<td>-$600,000</td>
<td>-$600,000</td>
</tr>
<tr>
<td>1</td>
<td>$15,000</td>
<td>0.94340</td>
<td>$14,151</td>
<td>-$585,849</td>
</tr>
<tr>
<td>2</td>
<td>$18,000</td>
<td>0.89000</td>
<td>$16,020</td>
<td>-$569,829</td>
</tr>
<tr>
<td>3</td>
<td>$21,600</td>
<td>0.83962</td>
<td>$18,136</td>
<td>-$551,693</td>
</tr>
<tr>
<td>4</td>
<td>$25,920</td>
<td>0.79209</td>
<td>$20,531</td>
<td>-$531,162</td>
</tr>
<tr>
<td>5</td>
<td>$31,104</td>
<td>0.74726</td>
<td>$23,243</td>
<td>-$507,920</td>
</tr>
<tr>
<td>6</td>
<td>$37,325</td>
<td>0.70496</td>
<td>$26,313</td>
<td>-$481,807</td>
</tr>
<tr>
<td>7</td>
<td>$44,790</td>
<td>0.66506</td>
<td>$29,788</td>
<td>-$451,819</td>
</tr>
<tr>
<td>8</td>
<td>$53,748</td>
<td>0.62741</td>
<td>$33,722</td>
<td>-$418,097</td>
</tr>
<tr>
<td>9</td>
<td>$64,467</td>
<td>0.59190</td>
<td>$38,176</td>
<td>-$379,921</td>
</tr>
<tr>
<td>10</td>
<td>$77,307</td>
<td>0.55839</td>
<td>$43,184</td>
<td>-$336,737</td>
</tr>
<tr>
<td>11</td>
<td>$92,876</td>
<td>0.52679</td>
<td>$48,926</td>
<td>-$287,778</td>
</tr>
<tr>
<td>12</td>
<td>$111,401</td>
<td>0.49697</td>
<td>$55,388</td>
<td>-$232,390</td>
</tr>
<tr>
<td>13</td>
<td>$133,742</td>
<td>0.46884</td>
<td>$62,703</td>
<td>-$189,687</td>
</tr>
<tr>
<td>14</td>
<td>$160,490</td>
<td>0.44230</td>
<td>$70,485</td>
<td>-$139,202</td>
</tr>
<tr>
<td>15</td>
<td>$192,588</td>
<td>0.41727</td>
<td>$78,360</td>
<td>-$80,842</td>
</tr>
<tr>
<td>16</td>
<td>$231,105</td>
<td>0.39385</td>
<td>$90,254</td>
<td>$72,932</td>
</tr>
</tbody>
</table>

Payback for B is 15.2 years

**Solution:**

SELECT A

**Self Study**

- See Ex 5.8 in Textbook p188 for to see the drawback of this method when comparing short-lived and longer-lived alternatives.
- Section 5.7 in Textbook p190—193, Life Cycle Cost
- Section 5.8 in Textbook p194—197, Present Worth of Bonds