Supply Chain Concepts: Double Marginalization and Risk Pooling

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This document is intended as a concise study review of two elementary concepts in supply chain management for Ph.D. students in ISEN who wish to prepare for the Production/Logistics/SCM area qualifying exams. This is *not* intended to be a rigorous derivation of these two concepts.

1 Double Marginalization

Consider a 2-stage supply chain consisting of one manufacturer and one retailer. The manufacturer produces a good at cost c, sells it to the retailer at $w > c$, and the retailer sells the good to the end consumer at price $r > w$. Assume demand is described by cdf (pdf) F (f) with mean $\mu$and standard deviation σ. Retailer and manufacturer make their own decisions (to maximize their profit), independently of each other. This is called a decentralized supply chain. Assuming a one-period, newsvendor-like setting, the retailer's optimal stocking level is given by $Q_d = F^{-1}(\frac{r-w}{r})$. The expected supply chain profit is given by $\Pi_d = r\mu - wQ_d + r\int_{Q_d}^{\infty}(Q_d - x)f(x)dx + (w-c)Q_d$.

Now assume that manufacturer and retailer are one company, with only one central decision maker. This is called a centralized supply chain. How does this change the dynamics? Now $w = c$, and hence the retailer's optimal stocking level is $Q_c = F^{-1}(\frac{r-c}{r})$. It is easy to see that $Q_c > Q_d$. Expected supply chain profit is now $\Pi_c = r\mu - wQ_c + r\int_{Q_c}^{\infty}(Q_c - x)f(x)dx$.

It turns out that $\Pi_c > \Pi_d$. This can easily be verified numerically. One can also show this analytically, but we will not do this here. Intuitively, why is $\Pi_c > \Pi_d$? The decentralized system's stocking decision is based on the manufacturer's markup $(w - c)$. Hence the retailer is going to stock less than the "overall" optimal amount. Hence the overall profit (which is a function of how many goods can be sold to the end consumer) is going to be lower. This influence of the manufacturer's markup is called double marginalization.

Ways to mitigate double marginalization: quantity forcing, wholesale price forcing, supply chain contracts.

2 Risk Pooling

Consider a single product that is stocked at N separate locations. Assume that demand for the product is a normal random variable D_i with known mean μ_i and standard deviation σ_i at each location, and that the D_i are independent of each other. If each location only serves its own demand, then location i needs to hold an amount of safety stock that allows it to hedge against the demand uncertainty associated with D_i. To simplify the scenario, we will again assume that this is a single-period, newsvendor type setting. Hence the safety stock at location i is $ss_i = \sigma_i z$, where z is the z-value that corresponds to some service level target. Therefore, the total safety stock across all locations is $ss = z\sum_{i=1}^{N}\sigma_i$.

Now consider the situation where we centralize all inventory holding at one location. Therefore this location needs to satisfy the demand $D = \sum_{i=1}^{N} D_i$. All else being equal, the safety stock at this single location is now $ss_c = z\sigma_{all}$, where σ_{all} is the standard deviation of D. Adding up the individual variances, we obtain $\sigma_{all} = \sqrt{\sum_{i=1}^{N}(\sigma_i)^2}$. Comparing ss and ss_c reveals that $ss_c \leq ss$, because $\sqrt{\sum_{i=1}^{N}(\sigma_i)^2} \leq \sum_{i=1}^{N}\sigma_i$ by the subadditivity property of the square root function over the non-negative reals.

Hence by consolidating inventory at a single location, the amount of safety stock necessary to ensure a given service level has decreased. The reason for this is that the standard deviation of the aggregate demand is lower than the sum of the standard deviations of the individual demands. This concept is called risk pooling.