ISEN 601
Location Logistics
Dr. Gary M. Gaukler
Fall 2011

Location on Networks

- Now consider Center problems
  - Center problem definition:
    \[ \min z \]
    \[ \text{subject to } d(y, u_i) \leq z, \quad \forall i \]

  - Minimax Problem

  - Coordinate from NF only have 1 in this model

Mathematical Formulation:

\[ \min z \]
\[ \text{subject to } d(y, u_i) \leq z, \quad \forall i \]

Number of existing facilities

1
Location on Networks

- Should a center always be located at a node?

\[ \text{NO} \]

\[ \text{NF max dist} = 2 \]

\[ v_1 \quad \text{↓} \quad v_2 \]

\[ x \quad 4 \]

2 Types

- Weighted
- Unweighted - \( d \) pure distance

1-Center On Unweighted Tree

[Diagram]

- \( W_i = 1 \)
- \( v_i \rightarrow \) pure distance

Step 1: Pick any node \( v \) on tree
Step 2: Find tip node \( v' \leftarrow \) farthest from \( v \)
Step 3: Find tip node \( v'' \leftarrow \) farthest from \( v' \)
- Path between \( v', v'' \) is the longest path on tree

Step 4: Find mid point & locate NF Facility there

Prove ?
Example

1.) Pick node \( V_3 \)
2.) \( V' = \) node \( V_1 \)
3.) \( V' = \) node \( V_2 \)
Total distance = 22

\[ \frac{22}{2} = 11 \]

1.) Locate NF between nodes 3 & 4.

1-Center on Weighted Tree

- Problem formulation:

\[
\begin{align*}
\min \ z \\
\text{s.t.} \quad w_i d_i(v_i) \leq z + y_i \\
& \quad \downarrow \\
& \quad d(v_i) \leq \frac{z}{w_i} v_i \\
& \quad d(v_i, v_j) \leq \frac{z}{w_j} \text{ if } j \\
& \quad d(v_i, v_j) \leq \frac{z}{w_i} + \frac{z}{w_j} \\
& \quad d(v_i) + d(v_j) \leq z + \frac{z}{w_i} + \frac{z}{w_j} \\
& \quad d(v_i, v_j) \leq \frac{z}{w_i} + \frac{z}{w_j} \\
& \quad b_{ij} = \frac{w_i w_j d(v_i, v_j)}{w_i + w_j} \\
& \quad b_{ij} \leq z \quad \text{if } i \neq j \\
& \quad b_{si} = \max \{ b_{ij} \mid i \leq j \leq 3 \} \\
& \quad b_{st} \leq z \quad \Rightarrow \text{optimal value}
\end{align*}
\]
1-Center on Weighted Tree

\[ b_{st} \leq 2 \quad \text{claim: } b_{st} \text{ optimal value for } z \]

\[ b_{st} = \max_{bij} = \max \frac{w_i w_j d(v_i, v_j)}{w_i + w_j} \]

\[ W_s d(y, v_s) = b_{st} \quad y \text{ is on the path } \]

\[ W_t d(y, v_t) = b_{st} \quad (v_s, v_t) \]

\[ d(y, v_s) = \frac{W_t}{W_s + W_t} d(v_s, v_t) \]

\[ d(y, v_t) = \frac{W_s}{W_s + W_t} d(v_s, v_t) \]
1-Center on Weighted Tree

q

1-Center on Weighted Tree
Summary of Algorithm

1) Calculate \( b_{ij} = \left[ \frac{\sum_{k} w_k d(v_i, v_j)}{w_i + w_j} \right] \) for \( i \neq j \)

2) \( b_{st} = \text{objective function value for } z \)

3) The optimal location \( y^* \) is on the shortest path from \( V_s \) to \( V_t \)

4) Calculate the length of the shortest path from \( V_s \) to \( V_t \)

5) \( a(y^*, V_s, V_t) = \frac{w_t}{w_s} d(V_s, V_t) \)

Algorithm Refinement

- Number of calculations necessary for bounds matrix: \( m \times \frac{m(m-1)}{2} \)

\( a, b, c, d \) are equal w/ \( a', b', c', d' \) respectively.

Only need to calculate upper triangle.
Algorithm Refinement

- It is possible to calculate the elements of matrix B "on the fly":
  - Choose any row r of the matrix
  - Calculate bij for row r and find the largest element in that row -> column c
  - Calculate bij for column c and find the largest element in that column -> row r
  - Continue until you find the same element in succession. This element is bst.

Example

\[
\begin{align*}
\text{b}_{16} &= \frac{(1)(1)}{(1+1)} = \frac{18}{2} = 9 \\
\text{b}_{26} &= \frac{(4)(1)}{(4+1)} = \frac{48}{5} = 9.6 \\
\text{b}_{36} &= \frac{(2)(1)}{(2+1)} = \frac{32}{3} = 10.67 \\
\text{b}_{46} &= \frac{(2)(1)}{(2+1)} = \frac{16}{3} = 5.33 \\
\text{b}_{56} &= \frac{(7)(1)}{(6+1)} = \frac{98}{7} = 14 \\
\end{align*}
\]

Choose column 6

\[
\begin{align*}
\text{b}_{14} &= \frac{(4)(1)}{(4+1)} = 4.8 \\
\text{b}_{24} &= \frac{(4)(2)}{(4+1)} = 5.33 \\
\text{b}_{25} &= \frac{(2)(1)}{(2+1)} = 5.33 \\
\text{b}_{35} &= \frac{(2)(1)}{(2+1)} = 5.33 \\
\end{align*}
\]

STOP
Compare with Unweighted Tree

- The algorithm for calculating the matrix elements is very similar to the 1-center solution algorithm for the unweighted tree case:

Exam 1 =D grade distribution >50% A

Unimodal distribution
≤85 look over for reasons