Chapter 6 – Annual Worth Analysis

INEN 303
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Chapter 6 – Annual Worth Analysis

- Advantages and Uses
- Calculation of Capital Recovery and AW Values
- Evaluating Alternatives by Annual Worth Analysis
- Annual-Worth of a Permanent Investment

Advantages and Uses

- Ideal approach for comparing alternatives with different lives under LCM assumptions
- *AW value has to be calculated for only one life cycle*
- *LCM comparison is implicit as,*
  $AW_{LCM} = AW_{Life}$
- Popular and easily understood
- Results are reported in $/time period

Capital Recovery and AW Value

- Capital Recovery is the **equivalent annual cost** of obtaining the asset plus the salvage
- CR is a function of {P, SV, i%, and n }
- AW is comprised of two components: capital recovery for the initial investment P at a stated interest rate (MARR) and the equivalent annual amount A

- An alternative usually has the following cash flow estimates:
  - **Initial Investment (P)** – the total first cost of all assets and services required to initiate the alternative.
  - **Salvage Value (SV)** – the terminal estimated value of assets at the end of their useful life.
  - **Annual Amount (A)** – the equivalent annual amount; typically this is the annual operating cost (AOC).

- Assume P, SV and A are just the magnitudes, to find CR:
  - **Method I : Compute AW of the original cost and add the AW of the salvage value**
    CR = - P(A|P, i, n) + SV(A|F, i, n)
  - **Method II : Add the present worth of the salvage value to the original cost, then compute the annual worth of the sum.**
    CR = [- P + SV(P|F, i, n)] (A|P, i, n)

AW = CR – A (Note the difference from the book)

- Example 6.1: A contractor purchased a used crane for $11,000. His operating cost will be $2700 per year, and he expects to sell it for $5000 five years from now. What is the equivalent annual worth of the crane at an interest rate of 10% ?

Solution:

CR = -11,000(A/P, 10%, 5) +5000(A/F,10%,5)

AW = -11,000(A/P, 10%, 5) +5000(A/F,10%,5) -2700

= -11,000(.2638) + 5000(.1638) – 2700

= -$4782.8

- Example 6.2: Calculate the AW for the following cash flow. Assume the MARR is 12% per year

	Year	Amount
Initial investment	0	8 million
Initial investment	1	5 million
Annual operating cost	1-8	0.9 million
Salvage value	8	0.5 million

First find the capital recovery (CR)

Method I:

CR = [-8.0 - 5.0(P/F,12%,1)](A/P,12%,8) + 0.5(A/F,12%,8)

= [-8.0-5.0*(.8929)](.2013) + 0.5*(.0813)

= $-2.47 million

Method II:

CR = [-8.0 - 5.0(P/F,12%,1) + 0.5(P/F,12%,8)](A/P,12%,8)

= [-8.0-5.0*(.8929) + 0.5*(.4039)](.2013)

= $-2.47 million

→ AW = CR - A

= -2.47 – 0.9 = $-3.37 million

Evaluating Alternatives by AW Analysis

- For mutually exclusive alternatives, calculate AW over one life cycle at the MARR
- One alternative: AW≥0, MARR is met or exceeded
- Two or more alternatives: Choose the alternative with numerically largest AW value
- Note that you are making a comparison over LCM to ensure equal service
- Your calculations are simplified since AW over LCM is the same as AW over life cycle

- Example 6.3: The following costs are estimated for two equal-service tomato-peeling machines to be evaluated by a canning plant manager.

	Machine A	Machine B
First Cost, $	26,000	36,000
Annual maintenance cost, $	800	300
Annual labor cost, $	11,000	7,000
Extra annual income taxes, $	-	2,600
Salvage value, $	2,000	3,000
Life, years	6	10

If the minimum required rate of return is 15% per year, help the manager decide which machine to select.

Solution:

Machine A:

AW_A = -26,000(A/P,15%,6) + 2,000 (A/F,15%,6) - 11,800
= -26,000*(.26424) + 2,000*(.11424) – 11,800
= $-18,442

Machine B:

AW_B = -36,000(A/P,15%,10) + 3,000 (A/F,15%,10) - 9,900
= -36,000*(.19925) + 3,000*(.04925) – 9,900
= $-16,925

Select machine B since $AW_B > AW_A$.

- Example 6.4: Assume the company in previous example is planning to exit the tomato canning business in 4 years. At that time, the company expects to sell machine A for $12,000 or machine B for $15,000. All other costs are expected to remain the same. Which machine should the company purchase under these conditions?

NOTE:

This is a study period problem. So we have considered all cash flows only for the study period (4 years).

Solution:

AW_A = -26,000(A/P,15%,4) +12,000 (A/F,15%,4) – 11,800
= -26,000*(.35027) + 12,000*(.20027) – 11,800
= $-18,504

AW_B = -36,000(A/P,15%,4) +15,000 (A/F,15%,4) – 9,900
= -36,000*(.35027) + 15,000*(.20027) – 9,900
= $-19,506

Select machine A as $AW_A > AW_B$.

Example 6.5:

- A public utility is trying to decide between two different sizes of pipe for a new water main. A 250-mm line will have an initial cost of $40,000, whereas a 300-mm line will cost $46,000. Since there is more head loss through the 250-mm pipe, the pumping cost for the smaller line is expected to be $2500 per year more than for the 300-mm line. If the pipes are expected to last for 15 years, which size should be selected if the interest rate is 12% per year? Use an annual-worth analysis.

Solution:

AW_{250} = -40,000(A/P, 12%, 15) – 2500
= -$8,373

AW_{300} = -46,000(A/P, 12%, 15)
= -$6,754

Select the 300 mm pipe

Reminder: Capitalized Cost (CC)

- Capitalized Cost (CC) for a uniform series A of end-of-period cash flows:

P=A(P/A, i, n)=A[$(1+i)^n$ – 1]/[i$(1+i)^n$] $= A\left[\frac{1 - \frac{1}{(1+i)^n}}{i}\right]$

$$\rightarrow \lim_{n \to \infty} A\left[\frac{1 - \frac{1}{(1+i)^n}}{i}\right] = A / i$$

Now, we have: CC = A/i

Also, A = CC(i)

Annual-Worth of a Permanent Investment

If an investment has infinite life, it is called a perpetual (permanent) investment. If P is the present worth of the cost of that investment, then AW is P times i.

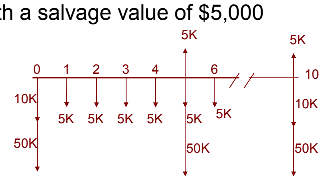
AW= P*i

Example 6.6:

Two alternatives are considered for covering a football field. The first is to plant natural grass and the second is to install AstroTurf. Interest rate is 10%. Cost structure for each alternative is given below.

Alternative I:

Natural Grass - Replanting will be required each 10 years at a cost of $10,000. Annual cost for maintenance is $5,000. Equipment must be purchased for $50,000 which will be replaced after 5 years with a salvage value of $5,000



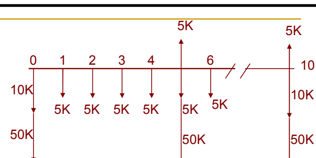
Alternative II:

AstroTurf - Installing AstroTurf cost $150,000 and it is expected to last indefinitely. Annual maintenance cost is expected to be $5,000

Solution:

AW of alt. A

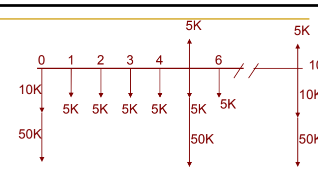
Cycle = 10 years



Planting: -10,000 (A|P, .10, 10) = $-1,628

1st Set Equipment (first 5 years):
[-50,000 + 5,000(P|F, .10, 5)] (A|P, .10, 10) = $-7,632

2nd Set of Equipment (second 5 years) :
{[-50,000 + 5,000(P|F, .10, 5)] (P|F, .10, 5)} (A|P, .10, 10) = $-4,739



AW of Alternative A, continued

Maintenance : -5,000 annually

Total : -1,628 - 7,632 - 4,739 - 5,000 = $-18,999

AW of Alternative B:

(AstroTurf - Installing AstroTurf cost $150,000 and it is expected to last indefinitely. Annual maintenance cost is expected to be $5,000)

Annual Cost of Installation : -150,000 (.10) = $-15,000
Maintenance: $-5,000 annually

Total : -15,000 - 5,000 = $-20,000

➔ Choose A

Example 6.7: Compare the following proposals to maintain a canal. Use interest rate 5%.

Proposal A (Buying Dredging Machine)	
First Cost, $	65,000
Annual maintenance cost, $	32,000
Salvage value, $	7,000
Life, years	10
Proposal B (Concrete Lining)	
Initial cost, $	650,000
Annual maintenance cost, $	1,000
Lining repairs every 5 years, $	1,800
Life, years	permanent

Solution:

AW_A = -65,000(A/P,5%,10)+7,000(A/F,5%,10) - - 32,000 = $- 39,861

AW_B = -650,000(0.05) – 1,000 - 1,800(A/F,5%,5) = $- 33,826

Choose proposal B.

Example 6.16

- The cash flow associated with a project having an infinite life is $-100,000 now, $-30,000 each year, and an additional $-50,000 every 5 years beginning 5 years from now. Determine its perpetual equivalent annual worth at an interest rate of 20% per year.

Solution

AW = -100,000(0.20) - 30,000 - 50,000(A/F,20%,5) = $-56,719 per year

Example 6.17

- A philanthropist working to set up a permanent endowment wants to deposit a uniform amount of money each year, starting now and for 10 more (11 deposits), so that $ 10 million per year will be available for research related to planetary colonization. If the first $10 million grant is to be awarded 11 years from now, what is the size of the uniform donations, if the fund will generate income at a rate of 15% per year?

Solution

First find P in year 10 for the $10 million annual amounts and then use the A/F factor to find A:

P_{10} = -10/0.15 = -$66.667 million

A = -66.667(A/F,15%,11)= -$2,738,000 per deposit

Example 6.18

- The costs associated with a certain robotic arm are $40,000 now and $24,000 per year, with a $6000 salvage value after 3 years. Determine the perpetual equivalent annual worth of the robot at an interest rate of 20% per year.

Solution

The perpetual uniform annual worth is the AW for one life cycle:

AW = -40,000(A/P,20%,3) - 24,000
+ 6000(A/F,20%,3) = $-41,341