Power and Rate Control for Cognitive Radios

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Abstract

In this report, the power and rate control schemes for cognitive radios (CRs) are considered, which operate over multiple frequency bands in the presence of licensed primary radios (PRs). Specifically, based on a delayed spectrum sensing information, an optimal algorithm to maximize the data rate for a single CR and a suboptimal algorithm to maximize the sum rate for multiple CRs are proposed with consideration of the CR-to-PR interference, respectively. In the proposed algorithms, the behavior of PRs is modeled as a two-state discrete-time Markov Chain (DTMC). Based on such a model, the power and rate control problems are formulated as chance constrained optimization problems and solved by dual decomposition methods. Simulation results show that the proposed algorithms lead to a significant performance improvement compared to heuristic algorithms.

I. INTRODUCTION AND LITERATURE REVIEW

The fast growth in wireless services makes the spectrum over-crowded since the government has to assign non-overlapping spectrum portions to most of the services. In year 2002, the Federal Communications Commission (FCC) Spectrum Policy Task Force published a report, indicating that there is a spectrum shortage for further licensing, while more than 90 percent of the already-licensed spectrum remains idle at a given time and location [1]. To explore the under-utilized spectrum resources, cognitive radio (CR) techniques have been proposed to implement opportunistic spectrum access in the licensed legacy bands [2]-[4].

One of the most important technical requirements for the CRs is that their data transmission should not result in harmful interference to primary radios (PRs) who are licensed users in the existing systems. To satisfy such a requirement, a CR performs spectrum sensing to detect the idle frequency bands, so-called spectrum holes, and adjusts its carrier frequency, transmit power, data rate, and other transmission parameters in a timely manner (e.g., packet by packet) [5]-[8]. Specifically, a decentralized scheme for CRs to maximize transmission rate with the consideration of spectrum sensing error is proposed in [5]. In [6], a distributed MAC protocol to maximize the data rate of CR links under an energy constraint is presented with the assumption of error-free spectrum sensing. A common aspect in all these schemes is the requirement of the instantaneous spectrum sensing output (e.g., busy or idle for a frequency band). Unfortunately, such environmental information may not be available at the beginning of each control time slot. Instead, a CR may only know the spectrum sensing results for previous time slots or some statistical knowledge about the PR behaviors. In this case, it is difficult for a CR to determine the transmission parameters in order to optimize certain performance, while keeping a tolerable interference to its surrounding PRs.

In this report, we consider the power and rate control schemes for CRs operating over multiple frequency bands. Based on a delayed spectrum sensing output and the statistical behavior of PRs, we propose an optimal power and rate control scheme to maximize the data rate for a single CR and a suboptimal scheme to maximize the sum rate for multiple CRs, while keeping
the CR-to-PR interference below a given level. To take advantage of the delayed spectrum sensing output, we model the PR occupancy of the channels as a discrete-time Markov chain (DTMC). Based on such a model, we formulate the power and rate control problems as chance constrained optimization problems and propose several algorithms to solve them by using dual decomposition methods. The proposed algorithms are then evaluated with numerical examples.

The rest of the report is organized as follows. The formal problem statement and formulation are described in Section II. The proposed algorithms are discussed in Section III. The computational experiments are shown in Section IV. Finally, Section V summarizes our conclusions and future work.

II. Formal Problem Statement

In this section, we will describe the power and rate control problems for CRs operating over multiple frequency bands (channels). We first consider a simple case where only one CR exists, then we will extend our formulation to the case of multiple CRs, who may interfere with each other.

A. Single CR Case

Consider a single CR transmitter-receiver pair operating over \( N \) channels, each of them with the same bandwidth \( W \). These \( N \) channels are licensed to a primary network whose users communicate according to a synchronous slotted structure. Over each time slot, the CR performs spectrum sensing by which the occupancy information of PR receivers could be learned. Since PR receivers only receive signals, it is difficult for the CR to detect the activity of a PR receiver in a real-time way. However, we assume that the CR can obtain the spectrum occupancy information for previous time slot. The wireless channels that the CR link undergoes is assumed to be block and frequency-selective fading, i.e., the channel gains are different for all channels at each slot; And the wireless channels remain unchanged within each time slot but vary over different time slots. Unlike the difficulty of the instantaneous spectrum sensing, the CR can easily estimate the real-time channel gains from the CR transmitter to the CR receiver by RTS/CTS mechanism. Based on the delayed spectrum sensing output and the instantaneous channel gains, the CR transmitter determines the channels to access and the corresponding transmit power and rate allocation. The resulting transmission parameters are then sent to CR receiver for data reception via a reliable control channel. In this case, we focus on design an optimal power and rate control strategy for the CR link to maximize its data rate, while keeping the CR-to-PR interference below a given level. Obviously, the power and rate control strategy for the CR is highly related to the PR behavior.

Assume that the PRs can access the \( N \) channels with absolute priority. According to the occupancy of PR receivers, each channel has two state to the CR: BUSY or IDLE. For convenience, define an indicator function \( I_n(t) \) for the channel \( n \) at time slot \( t \) as

\[
I_n(t) = \begin{cases} 
0 & \text{if the } n \text{th channel is BUSY at time } t \\
1 & \text{if the } n \text{th channel is IDLE at time } t \end{cases}
\]  

(1)

We assume that \( I_n(t) \)'s are constant within each time slot but varying over different time slots. Since the behavior of PRs is correlated in time\(^1\), we assume that the evolution of each channel independently follows a two-state DTMC as shown in Fig. 1, where \( \alpha \) is the transition probability from BUSY to IDLE and \( \beta \) is the transition probability from IDLE to BUSY. Note that

\(^1\)If a PR is active in the current slot, it is more likely to be active in the next slot; if it is inactive in the current slot, it is more likely to be inactive in the next slot. This is based on the assumption that the slot length is less than the coherence time of the PR activity random process.
we implicitly assume that \( \alpha < 0.5 \) and \( \beta < 0.5 \). Given \( \alpha \) and \( \beta \), the stationary distribution of the DTMC is given by

\[
[\pi_0, \pi_1] = \left[ \frac{\beta}{\alpha + \beta}, \frac{\alpha}{\alpha + \beta} \right]
\]  

where \( \pi_1 \) can be considered as the percentage of the time that the channel remains IDLE.

At time \( t \), if channel \( n \) is IDLE (\( I_n(t) = 1 \)), the CR can utilize it without consideration of the interference to PRs; Otherwise, we have to constrain the transmit power of the CR such that the resulting interference to the PR receiver is below a tolerable level \( \gamma \). Specifically, let \( p_n(t) \) and \( c_n(t) \) denote the transmit power of the CR and the interference coefficient from the CR transmitter to the PR receiver for channel \( n \), respectively. When \( I_n(t) = 0 \), \( p_n(t) \) has to satisfy the following constraint

\[
p_n(t)c_n(t) \leq \gamma
\]  

Unfortunately, the CR has no knowledge of the instantaneous values of \( I_n(t) \) and \( c_n(t) \). Therefore, we cannot guarantee that (3) is always satisfied. Define a CR-to-PR disturbance probability as the probability that (3) is not satisfied. Based on the stochastic characteristics of \( I_n(t) \) and \( c_n(t) \), we can adjust \( p_n(t) \) such that the CR-to-PR disturbance probability is below a predefined threshold \( p_0 \), i.e.,

\[
\Pr\{p_n(t)c_n(t) > \gamma | I_n(t) = 0\} \leq p_0.
\]  

Assuming \( c_n(t) \) is subject to exponential distribution with parameter \( \lambda \), given the knowledge of the previous indicator values \( I_n(t-1)'s \), we transform (4) into

\[
p_n(t) \leq -\frac{\lambda \gamma}{\log(1 - ap_0)}
\]  

where \( a \) is given by

\[
a = \begin{cases} 
1 - \alpha & \text{if } I_n(t-1) = 0 \\
\beta & \text{if } I_n(t-1) = 1 
\end{cases}
\]

Given the knowledge of the previous indicator values and the instantaneous channel gains, our objective is to maximize the data rate of the CR, while satisfying the total transmit power constraint \( P \) and the chance constraint (4), i.e.,

\[
\max \sum_{n=1}^{N} r_n(t) \\
\text{s.t.} \sum_{n=1}^{N} p_n(t) \leq P, \quad \Pr\{p_n(t)c_n(t) > \gamma | I_n(t) = 0\} \leq p_0, \quad n = 1, 2, \ldots, N,
\]

We assume that each channel can only be occupied by one PR whose receiver is located in the neighborhood of the CR at each time slot. If there are more than one PRs on each channel, we can formulate the problem by adding more probabilistic constraints. Our results can be easily extended to this case.
where $r_n(t)$ is the transmission rate for the CR on the channel $n$, which can be expressed as

$$r_n(t) = W \log \left( 1 + \frac{p_n(t)h_n(t)}{\sigma^2} \right)$$

(8)

where $h_n(t)$ is the channel gain for channel $n$ and $\sigma^2$ is the variance of the Gaussian noise-plus-interference for each time slot. Note that we assume the noise-plus-interference process as a stationary Gaussian process given the possible large number of in-band interferers3. Note that since the chance constraint (4) is equivalent to the linear constraint (5), (7) is a convex optimization problem, which can be easily solved by some standard optimization methods (e.g., interior point method) [9] when $N$ is a small value. When $N$ becomes large, the computational complexity is still high. As shown in Section III, a low-complexity algorithm based on dual decomposition methods can be used to obtain the optimal solution of (7).

B. Multi-CR Case

In this section, we will describe the problem formulation for the multi-CR case, where there are $K$ CRs in a neighborhood. We assume that there is a center control node in the neighborhood who coordinates the data transmission of the CRs. Over each time slot, each CR performs spectrum sensing and channel estimation and then sends the delayed spectrum information and instantaneous channel information to the control node through a reliable control channel. Based on such information from all CRs, the control node calculates the transmit power and rate allocation for each CR. Such transmission parameters are sent to each CR transmitter-receiver pair for opportunistic communications. We assume that the channel fading for each CR is independent of each other. Based on the same model of the PR behavior as in single CR case, we focus on designing an efficient power and rate control strategies for the CRs to maximize their sum rate, while keeping the CR-to-PR interference below a given level.

To make the problem tractable, we assume that each CR can use multiple channels but each channel can only be used by one CR exclusively at each time slot. Let $I_n^k(t)$ and $p_n^k(t)$ denote the indicator value and transmit power on channel $n$ for user $k$ at time $t$, respectively. The sum-rate maximization problem can be expressed as

$$\max \sum_{k=1}^{K} \sum_{n=1}^{N} w_n^k(t) r_n^k(t)$$

(9)

s.t. $\sum_{n=1}^{N} p_n^k(t) \leq P_k$, $k = 1, 2, \ldots, K$,

$\sum_{k=1}^{K} w_n^k(t) \leq 1$, $n = 1, 2, \ldots, N$,

$$\Pr \left\{ \sum_{k=1}^{K} w_n^k(t)p_n^k(t)c_n^k(t) > \gamma | I_n^k(t) = 0 \right\} \leq p_0$$, $n = 1, 2, \ldots, N$,

where $w_n^k(t)$’s are binary design variables with $w_n^k(t) = 1$ indicating that CR $k$ uses channel $n$ at time $t$; $\tilde{k}$ is chosen such that $w_n^{\tilde{k}}(t) = 1$; $P_k$ is the total power constraint for CR $k$; $c_n^k(t)$ is the interference coefficient from the transmitter of CR $k$ to the PR receiver for channel $n$ at time $t$, which is assumed to be exponential distributed with parameter $\lambda_k$; and $r_n^k(t)$ is the transmission rate for the CR $k$ on channel $n$ at time $t$, which can be expressed as

$$r_n^k(t) = W \log \left( 1 + \frac{p_n^k(t)h_n^k(t)}{\sigma^2} \right)$$

(10)

The interferers include the PRs and other CRs both inside and outside the neighborhood.
where $h_n^k(t)$ is the channel gain on the channel $n$ for CR $k$. Note that the optimization problem (9) is a combinatorial problem, which is generally NP hard. Furthermore, we cannot get the explicit expression for the chance constraint in (9), which makes it even harder to solve. In Section III, we will propose a suboptimal algorithm to solve this problem by using dual decomposition methods.

### III. Solution Approach

In this section, we will describe the proposed algorithms to solve (7) and (9), respectively. First, we will show that the optimal solution of (7) has a closed-form expression. Afterwards, a suboptimal algorithm is proposed to solve (9).

**A. Single CR Case**

We use multipliers $\mu$ to dualize the first constraint in (7). This yields the following dual function

$$d(\mu) = \max \sum_{n=1}^{N} r_n(t) - \mu \left( \sum_{n=1}^{N} p_n(t) - P \right) \quad (11)$$

$$= \mu P + \sum_{n=1}^{N} \max (r_n(t) - \mu p_n(t)).$$

In this way, we decompose the original maximization problem into $N$ sub maximization problem, with the $n$th subproblem given by

$$\max \quad d_n(\mu) = r_n(t) - \mu p_n(t) \quad (12)$$

$$\text{s.t.} \quad p_n(t) \leq -\frac{\lambda \gamma}{\log(1 - ap_0)}.$$

Given $\mu$, we take the derivative of the objective function $d_n(\mu)$ with respect to $p_n(t)$, which can be expressed as

$$\frac{\partial d_n(\mu)}{\partial p_n(t)} = \frac{h_n(t)}{\sigma^2 + p_n(t)h_n(t)} - \mu. \quad (13)$$

Therefore, the optimal solution of (12) is given by

$$p_n(t) = \min \left\{ \left( \frac{1}{\mu} - \frac{\sigma^2}{h_n(t)} \right)^+, -\frac{\lambda \gamma}{\log(1 - ap_0)} \right\} \quad (14)$$

where $(x)^+ = \max(0, x)$. As such, given arbitrary $\mu$, we can solve the $N$ subproblems and then get the optimal objective value of (11).

The dual minimization problem is given by

$$\min \quad d(\mu) \quad (15)$$

$$\text{s.t.} \quad \mu \geq 0.$$

A subgradient of $d(\mu)$ can be expressed as

$$\nabla d(\mu) = P - \sum_{n=1}^{N} p_n(t). \quad (16)$$

We see that the subgradient corresponds to the unused power of the CR. Since the duality gap between the primal problem and the dual problem is zero for a convex problem. We can solve the primal problem (7) by solving the dual problem (15).

The standard dual descent method for the dual can be stated as follows:

- **Step 0**: Initialize $\nu = 0$ and $\mu^\nu = 1$. Pick a tolerance $\varepsilon > 0$ and a step size $\delta > 0$. 
• **Step 1:** Solve the subproblems given by (12), and get the optimal solution \( p_n^k(t) \) according to (14).

• **Step 2:** Update \( \mu^{\nu+1} \) according to

\[
\mu^{\nu+1} = \left( \mu^\nu - \delta \left( P - \sum_{n=1}^{N} p_n^k(t) \right) \right)^+. \tag{17}
\]

• **Step 3:** Stop if \( \|\mu^{\nu+1} - \mu^\nu\| \leq \varepsilon \). Otherwise, Set \( \nu = \nu + 1 \) and go to **Step 1**.

From standard convergence analysis of dual descent algorithms, we argue that if step size \( \delta \) is chosen sufficiently small, the objective value of \( d(\mu^\nu) \) decreases monotonically. Moreover, the corresponding \( \{p_n^k(t)\} \) is asymptotically satisfying the total power constraint of the CR.

We can also find the optimal solution of (15) by standard bisection searching method, which can be stated as follows:

• **Step 0:** Initialize \( \mu^{\text{min}} = 0 \) and \( \mu^{\text{max}} = 1 \). Pick a tolerance \( \varepsilon > 0 \).

• **Step 1:** Solve the subproblems given by (12) for \( \mu = \mu^{\text{max}} \). If \( \sum_{n=1}^{N} p_n(t) > P \), let \( \mu^{\text{max}} = 2\mu^{\text{max}} \), go to **Step 1**; otherwise, go to **Step 2**.

• **Step 2:** Repeat the following procedure until convergence.

1. Let \( \mu = \left( \frac{\mu^{\text{min}} + \mu^{\text{max}}}{2} \right) \).
2. Solve the subproblems given by (12) for the updated \( \mu \);
3. If \( \sum_{n=1}^{N} p_n(t) > P \), then \( \mu^{\text{min}} = \mu \), else \( \mu^{\text{max}} = \mu \).

Note that the dual decent method and the bisection method convergence to the same optimal solution of (15). The only difference is convergence speed. The convergence speed of the dual decent method depends on the step size \( \delta \), while the convergence speed of the bisection method depends on the initial value of \( \mu^{\text{max}} \).

**B. Multi-CR case**

In the case where multiple CRs coexist in a neighborhood, we have to find the optimal solution of (9). Similar to the dual decomposition method we use in single CR case, we use multipliers \( \bar{\mu} = [\mu_1, \mu_2, \ldots, \mu_K] \) to dualize the total power constraint for each CR in (9). This yields the following dual function

\[
d(\bar{\mu}) = \max \sum_{k=1}^{K} \sum_{n=1}^{N} \mu_k \left( \sum_{n=1}^{N} p_n^k(t) - P_k \right) - \sum_{k=1}^{K} \mu_k \left( \sum_{n=1}^{N} p_n^k(t) - P_k \right) \tag{18}
\]

\[
= \sum_{k=1}^{K} \mu_k P_k + \max \sum_{k=1}^{K} \sum_{n=1}^{N} \left( w_n^k(t) r_n^k(t) - \mu_k p_n^k(t) \right) \]

\[
= \sum_{k=1}^{K} \mu_k P_k + \max \sum_{n=1}^{N} \sum_{k=1}^{K} \left( w_n^k(t) r_n^k(t) - \mu_k p_n^k(t) \right) \]

In this way, we decompose the original maximization problem into \( N \) sub maximization problem, with the \( n \)th subproblem given by

\[
\max \quad d_n(\bar{\mu}) = \sum_{k=1}^{K} \left( w_n^k(t) r_n^k(t) - \mu_k p_n^k(t) \right) \tag{19}
\]

s.t.

\[
\sum_{k=1}^{K} w_n^k(t) \leq 1,\]

\[
\Pr \left\{ \sum_{k=1}^{K} w_n^k(t) p_n^k(t) c_n^k(t) > \gamma | I_n^k(t) = 0 \right\} \leq p_0.
\]
The maximization problem (19) can be solved by allocating channel \( n \) to the user which can provide the maximum objective value of the following optimization problem

\[
d_k^\mu(n) = \max \ r_k^\mu(n) - \mu_k p_k^\mu(n) \\
\text{s.t.} \quad p_k^\mu(n) \leq -\frac{\lambda_k \gamma}{\log(1 - \alpha_k p_0)}
\]

where \( a_k \) is given by

\[
a_k = \begin{cases} 
1 - \alpha & \text{if } I_k^\mu(n - 1) = 0 \\
\beta & \text{if } I_k^\mu(n - 1) = 1 
\end{cases}
\]

(21)

Similar to the optimal solution of (12), the optimal solution of (20) is given by

\[
p_k^\mu(n)(t) = \min \left\{ \left( \frac{1}{\mu_k} - \frac{\sigma^2}{h_k^\mu(n)(t)} \right)^+, -\frac{\lambda_k \gamma}{\log(1 - \alpha_k p_0)} \right\}.
\]

(22)

Let \( \hat{k} = \arg\max_k d_k^\mu(n) \). The optimal solution of (19) can be expressed as

\[
p_k^\mu(n)(t) = \begin{cases} 
0 & \text{if } k \neq \hat{k} \\
\min \left\{ \left( \frac{1}{\mu_k} - \frac{\sigma^2}{h_k^\mu(n)(t)} \right)^+, -\frac{\lambda_k \gamma}{\log(1 - \alpha_k p_0)} \right\} & \text{if } k = \hat{k}
\end{cases}
\]

(23)

and

\[
w_k^\mu(n)(t) = \begin{cases} 
0 & \text{if } k \neq \hat{k} \\
1 & \text{if } k = \hat{k}
\end{cases}
\]

(24)

The dual minimization problem is given by

\[
\min \quad d(\bar{\mu}) \\
\text{s.t.} \quad \bar{\mu} \geq 0.
\]

(25)

A subgradient of \( d(\mu) \) can be expressed as

\[
\nabla d(\bar{\mu}) = \left[ P_1 - \sum_{n=1}^N p_1^\mu(n)(t), P_2 - \sum_{n=1}^N p_2^\mu(n)(t), \ldots, P_K - \sum_{n=1}^N p_K^\mu(n)(t) \right]^T.
\]

(26)

Note that since (9) is not a convex problem, the duality gap between (9) and (25) is not zero, which means we can only obtain a upper bound of (9) by solving (25). However, in [10], it is shown that when \( N \) approaches infinity, the duality gap is asymptotically to be zero. Therefore, we can obtain an asymptotically feasible solution of (9) by solving (25) when \( N \) is a large number. The standard dual descent method for this problem can now be stated as follows:

- **Step 0:** Initialize \( \nu = 0 \) and \( \mu_\nu^k = 1 \) for \( k = 1, 2, \ldots, K \). Pick a tolerance \( \varepsilon > 0 \) and a step size \( \delta > 0 \).
- **Step 1:** Solve the subproblems given by (19), and get the optimal solution \( p_k^{\nu,\nu}(n)(t) \) according to (23).
- **Step 2:** Update \( \mu_\nu^{\nu+1} \) according to

\[
\mu_\nu^{\nu+1} = \mu_\nu^\nu - \delta \left( P_k - \sum_{n=1}^N p_k^{\nu,\nu}(n)(t) \right)^+.
\]

(27)

- **Step 3:** Go to **Step 4** if \( \|\mu_\nu^{\nu+1} - \mu_\nu^\nu\| \leq \varepsilon \). Otherwise, Set \( \nu = \nu + 1 \) and go to **Step 1**.
- **Step 4:** Allocate the channels to the CRs based on \( w_k^\nu(n)(t) \)'s obtained in **Step 3**, and perform the optimal single CR power and rate control algorithm for each CR.
When the number of CRs is small, we can also use bisection algorithm to efficiently solve problem (25). The bisection algorithm for two CRs is stated as follows, the extension to the case of more than two CRs is straightforward.

- **Step 0**: Initialize $\mu_{1}^{\min} = 0$ and $\mu_{1}^{\max} = 1$.
- **Step 1**: Find the optimal $\mu_{2}$ and the corresponding optimal solution $\{p_{n}^{k}(t)\}$ for $\mu_{1} = \mu_{1}^{\max}$. If $\sum_{n=1}^{N} p_{n}^{1}(t) > P_{1}$, let $\mu_{1}^{\max} = 2\mu_{1}^{\max}$, go to **Step 1**; Otherwise, go to **Step 2**.
- **Step 2**: Repeat the following procedure until convergence.
  1. Let $\mu_{1} = \frac{\mu_{1}^{\min} + \mu_{1}^{\max}}{2}$;
  2. Find the optimal $\mu_{2}$ and the corresponding optimal solution $\{p_{n}^{k}(t)\}$ for the updated $\mu_{1}$;
  3. If $\sum_{n=1}^{N} p_{n}^{1}(t) > P_{1}$, then $\mu_{1}^{\min} = \mu_{1}$, else $\mu_{1}^{\max} = \mu_{1}$.

- **Step 3**: Allocate the channels to the CRs based on $w_{k}^{n}(t)$’s obtained in **Step 2**, and perform the optimal single CR power and rate control algorithm for each CR.

The procedure to find the optimal $\mu_{2}$ and the corresponding optimal solution $\{p_{n}^{k}(t)\}$ and $w_{k}^{n}(t)$ for a fixed $\mu_{1}$ is given as follows:

- **Step 0**: Initialize $\mu_{2}^{\min} = 0$ and $\mu_{2}^{\max} = 1$.
- **Step 1**: Solve the subproblems given by (19) for $\mu_{2} = \mu_{2}^{\max}$. If $\sum_{n=1}^{N} p_{n}^{2}(t) > P_{2}$, let $\mu_{2}^{\max} = 2\mu_{2}^{\max}$, go to **Step 1**; Otherwise, go to **Step 2**.
- **Step 2**: Repeat the following procedure until convergence.
  1. Let $\mu_{2} = \frac{\mu_{2}^{\min} + \mu_{2}^{\max}}{2}$;
  2. Solve the subproblems given by (19) for the updated $\mu_{2}$;
  3. If $\sum_{n=1}^{N} p_{n}^{2}(t) > P_{2}$, then $\mu_{2}^{\min} = \mu_{2}$, else $\mu_{2}^{\max} = \mu_{2}$.

The bisection searching algorithm works more efficient than the dual decent algorithm especially when the number of CRs is small. When the number of CRs becomes larger, the dual decent algorithm is more efficient by appropriately selecting step size $\delta$.

### IV. Computational Experiment

In this section, we evaluate the performance of the proposed algorithms and compare them to some reference algorithms. For single CR case, we will evaluate the performance of the proposed algorithm in term of data rate and the CR-to-PR disturbance probability, respectively. For multi-CR case, we will evaluate the performance of the proposed algorithm in term of the sum rate of CRs and the CR-to-PR disturbance probability, respectively.

#### A. Single CR Case

In this case, we compare the proposed algorithm to a reference algorithm, in which the CR-to-PR interference is not taken into account. Specifically, the reference algorithm solves the following problem

$$\max \sum_{n=1}^{N} r_{n}(t) \quad \text{(28)}$$

subject to

$$\sum_{n=1}^{N} p_{n}(t) \leq P.$$
TABLE I
SYSTEM PARAMETERS

<table>
<thead>
<tr>
<th>System Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1</td>
</tr>
</tbody>
</table>

In the simulation, we set $N=256$, $W=1$, and $p_0=0.1$. The other system parameters are given in TABLE I.

In Fig.2, the performance of the proposed algorithm compared to the reference algorithm in term of data rate is drawn versus different values of signal-to-noise-plus-interference ratios (SINRs). The SINR here denotes the ratio of the CR transmit power to the noise plus interference for each channel on average. We see that in both algorithms, the data rate increases with the growing SINRs. The reference algorithm has the same performance for different PR behavior patterns (different values of $\alpha$ and $\beta$), which is better than that of the proposed algorithm. This is because the consideration of CR-to-PR interference in the proposed algorithm leads to some data rate loss. Furthermore, the proposed algorithm can achieve higher rate when the ratio of $\alpha$ to $\beta$ increases, since more IDLE channels the CR can use with higher ratio of $\alpha$ to $\beta$ at each time slot.

In Fig. 3, we compare the performance of the proposed algorithm and the reference algorithm in term of CR-to-PR disturbance probability versus different SINRs. We see that in both algorithms, the disturbance probability increases with the growing SINRs and with the decreasing ratio $\alpha$ to $\beta$. Lower ratio of $\alpha$ to $\beta$ means lower percentage of IDLE time for each channel, which will increase the CR-to-PR disturbance probability. Since there is no consideration of CR-to-PR interference in reference algorithm, the disturbance probability is above the given threshold $p_0$ in the interested SINR range. The proposed algorithm has significant performance improvement in term of CR-to-PR disturbance probability compared to the reference algorithm. We can see that the disturbance probability of the proposed algorithm is always roughly below the given threshold. Theoretically, the proposed
algorithm can guarantee that the CR-to-PR disturbance probability is below the given threshold, the slight violation in the figure is due to the inaccuracy of the simulation.

B. Multi-CR Case

In multi-CR case, we compare the proposed algorithm to two reference algorithms: \textit{reference algorithm 1} and \textit{reference algorithm 2}. In \textit{reference algorithm 1}, we fix the channel allocation (allocation the same number of channels to each CR for fairness) and perform the single optimal control and rate control algorithm for each CR at each time slot. In \textit{reference algorithm 2}, we maximize the sum rate of CRs without consideration of the CR-to-PR interference, i.e., we focus on the following optimization problem

$$\max \sum_{k=1}^{K} \sum_{n=1}^{N} w_{n}^{k}(t)r_{n}^{k}(t)$$

s.t. \quad \sum_{n=1}^{N} p_{n}^{k}(t) \leq P_{k}, \quad k = 1, 2, \ldots, K,$$

$$\sum_{k=1}^{K} w_{n}^{k}(t) \leq 1, \quad n = 1, 2, \ldots, N.$$  

In [10], a suboptimal algorithm is proposed to solve (29) based on dual decomposition, which we can adopt here as \textit{reference algorithm 2}. In the simulation, we consider two CRs in a neighborhood operating over 512 channels, each of them has the same system parameters as given in TABLE I. In addition, let $p_{0} = 0.1$.

In Fig. 4, the performance of the proposed algorithm compared to the two reference algorithms in term of sum rate is shown versus different values of signal-to-noise-plus-interference ratios (SINRs). The SINR here denotes the ratio of the transmit
power to the noise plus interference for each channel on average for each CR (all CRs have the same SINRs). We see that both proposed algorithm and reference algorithm 1 have higher data rate with increasing SINRs and decreasing ratio of $\alpha$ to $\beta$. The performance of the proposed algorithm is better than that of reference algorithm 1 for different PR behavior patterns. Since reference algorithm 2 does not consider CR-to-PR interference, it has the same performance for different PR behavior patterns. When $\alpha = \beta = 0.1$, the performance of the proposed algorithm is better than reference algorithm 2 in low SINR regime and worse in high SINR regime. When $\alpha = 0.2$ and $\beta = 0.1$, the proposed algorithm outperforms reference algorithm 2 over the whole SINR region.

In Fig. 5, we compare the performance of the proposed algorithm to the two reference algorithms in term of CR-to-PR disturbance probability. We see that disturbance probabilities of the proposed algorithm and reference algorithm 1 are roughly below the given threshold, while that of reference algorithm 2 exceeds the threshold dramatically.

V. CONCLUSIONS AND FUTURE WORK

In this report, we proposed an optimal power and rate control algorithm to maximize the data rate for a single CR and a suboptimal algorithm to maximize the sum rate for multiple CRs based on dual decomposition methods. The obtained strategies explores the time correlation and the statistical property of the PR behavior. Analytical and simulation results show that the proposed algorithms lead to a significant performance improvement against some heuristic algorithms. In the future work, we will evaluate the performance gap between our result and the optimal solution for multi-CR case and try to figure out other algorithms which may be more close to the optimal.

REFERENCES

Fig. 5. CR-to-PR disturbance comparison for multi-CR case.


